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STOCHASTIC RESPONSE WITH BIFURCATIONS TO NON-LINEAR DUFFING'S OSCILLATOR

Some classes of non-linear Fokker-Planck-Kolmogorov equations with bifurcating solutions have recently appeared in the literature [1, 2]. However, these equations are characterized by unusual drift forces and diffusion coefficients. In what follows here it will be shown that a response with bifurcation is possible for a non-linear Duffing oscillator, and that the probability density function of this response can be calculated numerically. This type of distribution function is characterized by multiple maxima and tends to be intrinsically non-Gaussian.

Consider the non-linear Duffing oscillator,

$$\ddot{y}+k\dot{y}+ay+by^3=\varepsilon x, \tag{1}$$

where k is the (positive) damping factor, a and b are linear and non-linear stiffness coefficients, ε is a constant, and x is a stochastic process given by the filter equation

$$\ddot{x}+l\dot{x}+x=n(t), \tag{2}$$

where l is the damping factor and $n(t)$ is white noise with $\langle n(t)\rangle=0$ and $\langle n(t)n(t+\tau)\rangle= D\,\delta(t)$; D is constant and $\langle\cdot\rangle$ indicates the ensemble average.

After the transformation $x_1=y$, $x_2=\dot{y}$, $x_3=x$ and $x_4=\dot{x}$, equations (1) and (2) have the form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}x_2 & 0 & 0 & 0\\ -ax_1-kx_2+x_3 & 0 & 0 & -bx_1^3\\ 0 & 0 & x_4 & 0\\ 0 & -x_3-lx_4 & 0 & 0\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ n(t)\end{bmatrix}. \tag{3}$$

With the probability density function denoted by $P(x_1, x_2, x_3, x_4/x_{10}, x_{20}, x_{30}, x_{40}, t)$ where x_{10}, x_{20}, x_{30} and x_{40} are the initial conditions, the Fokker-Planck-Kolmogorov

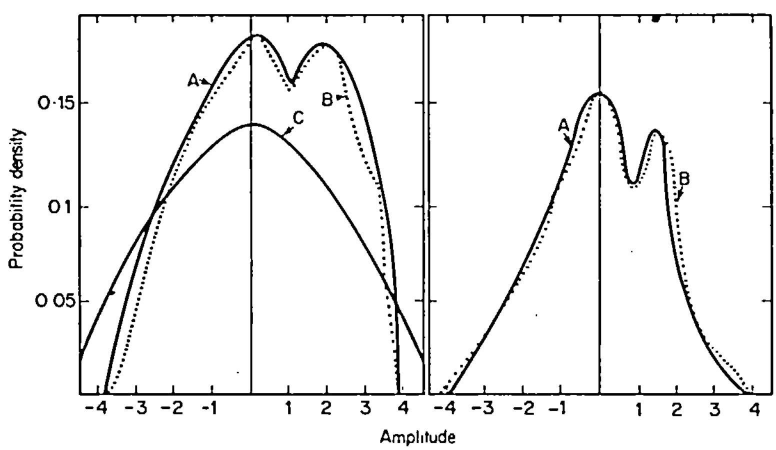

Figure 1. Amplitude probability density function $l=0{\cdot}2$, $\varepsilon=50$, $D=1$. A Path-integral solution; B, digital Monte-Carlo simulation; C, polynomial approximation. (a) $a=0{\cdot}94$, $b=3{\cdot}5$; (b) $a=0{\cdot}3$, $b=6{\cdot}0$.

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equation satisfying equation (3) becomes

$$\frac{\partial P}{\partial t}=-\frac{\partial}{\partial x_1}[x_2P]-\frac{\partial}{\partial x_2}[(-a-kx_2+\varepsilon x_3-bx_1^3)P]+\frac{\partial}{\partial x_3}[x_4P]$$
$$-\frac{\partial}{\partial x_4}[(-x_3-lx_4)P]+\frac{D}{2}\frac{\partial^2 P}{\partial x_4^2}. \qquad (4)$$

This equation for the steady state was solved numerically by the path-integral method [3]. The probability density functions obtained are shown in Figure 1(a) and (b). These probability density functions were compared with the results obtained by digital Monte-Carlo simulation. The same example as in Figure 1(a) was investigated by Tagata [4] who compared the result of the digital simulation with that from polynomial approximation, which does not produce the concave region in the amplitude probability density characteristic. The method of path-integral solution of the Fokker-Plank-Kolmogorov equation (4) gives the solution process with bifurcation, which explains the jump phenomena observed in real non-linear systems.

In a future paper the author hopes to present the complete bifurcation diagram for such a non-linear system.

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