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A SIMPLE MODEL OF A HUMAN RESPONSE TO A SEQUENCE OF RANDOM COLLISIONS

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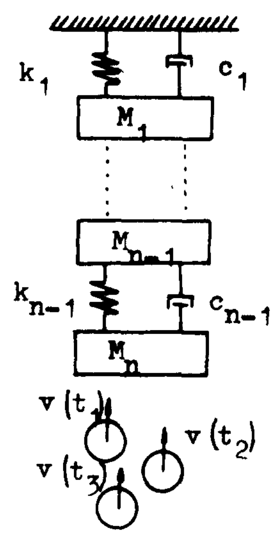
Introduction

The operator of any kind of percussive machine is subjected to reactions of very short duration in comparison with the periods of free vibrations of the human elements involved (for example the hand-arm system). These reactions can be described by the sequence of impulses, excited by the collisions of the human model element M_n with the objects of equal mass m, having random velocity v and random arrival time t_i -Fig. 1
The force which excites vibration is given in the following form [1, 2]:

$$F_n(t) = \frac{M_n m}{M_n + m}(1+k)\sum_i \left(v(t_i) - \dot{x}_{n0}\right)\delta(t-t_i)$$

where : $t \in [0, \infty)$ - time , k - restitution coefficient of collision, $\dot{x}_{n0}$ - initial velocity of the element M_n.
In this paper it was assumed that the velocities $v(t_i)$ exhibit a given distribution function $\mathcal{F}$ and that the arrival times t_i satisfy the Poisson distribution.

Fig. 1: Discrete model of a percussive machine--man system.



General solution

The vibrations excited by the sequence of random collisions of the n-degree of freedom dynamic system can be described by the following matrix equation:

$$\ddot{X}(\omega, t) + 2\beta \dot{X}(\omega,t) + \alpha X(\omega,t) = \varphi(\omega, t) \tag{1}$$

where: $\beta = \frac{1}{2} M^{-1}B$, $\alpha = M^{-1}C$, $\varphi(\omega,t) = \mathrm{col}[0,0,\dots,M_n^{-1}F_n(t)]$, $M = \mathrm{diag}[M_j]$ - n x n dimensional inertia matrix, $B = [b_{j,k}]$ - n x n dimensional damping coefficients matrix , $C = [c_{j,k}]$ - n x n dimensional stiffness matrix , $X(\omega, t) = \mathrm{col}[x_j(\omega, t)]$ - n x 1 dimensional vector of coordinates, $\omega \in \Omega$ and $(\Omega, \mathcal{B}, \mu)$ - probabilistic space , $j,k = 1,2,\dots,n$, Ω , $\mathcal{B}$ and μ are respectively the set of elementary events, the σ-field of its Borel subsets, the probabilistic measure.
By putting new coordinates : $X^{(1)} = X$, $X^{(2)} = \dot{X}$ we obtained the equation of vibrations of our system in the form of the system of Ito's stochastic differential equations:

$$\begin{aligned}
dx_1^{(1)} &= x_1^{(2)}\,dt\\
dx_1^{(2)} &= -\sum_{j=1}^{n}\left(2\beta_{1j}x_j^{(2)} - \alpha_{1j}x_j^{(1)}\right)dt\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
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dx_n^{(1)} &= x_n^{(2)}\,dt\\
dx_n^{(2)} &= -\sum_{j=1}^{n}\left(2\beta_{nj}x_j^{(2)} - \alpha_{nj}x_j^{(1)}\right)dt + \frac{(1+k)\,m}{m+M}\int_{-\infty}^{\infty}(v - \dot{x}_{n0}^{(2)})\,\mu(dt,dv)
\end{aligned} \tag{2}$$

where: μ is the random measure of the Poisson processes of velocity jumps. The values of these jumps fall in the interval $(v, v+dv)$ and the arrival time in the interval $(t, t+dt)$. The measure μ gives the number of such jumps and the parameters v describe the value of these jumps. The mean value of the measure μ is described by the distribution function $\mathcal{F}$.
By putting $EX^{(1)} = m^{(1)} = \mathrm{col}[m_j^{(1)}]$ and $EX^{(2)} = m^{(2)} = \mathrm{col}[m_j^{(2)}]$ (where

E - symbol of mean value) and making use of an averaging procedure, we obtain the system of equations for the first order moments of $X^{(1)}(\omega, t)$ and $X^{(2)}(\omega, t)$.
The second order moments : $\varkappa^{(o,s)}_{j,k} = E\left(x^{(o)}_j\right)^p \left(x^{(s)}_k\right)^r$; o , s = 1,2 ; p , r = 0,1,2 ; p+r = 2 and the third order moments: $\psi^{(o,s,z)}_{j,k,l} = E\left(x^{(o)}_j\right)^u \left(x^{(s)}_k\right)^w \left(x^{(z)}_l\right)^y$; o , s , z = 1,2 ; u , w , y = 1,2,3 ; u+w+y=3 can be obtained from Ito's formula [3]:

$$dV = \left(\frac{\partial V}{\partial t} + \sum_{i=1}^{2n} \frac{\partial V}{\partial x_i} G_i\right) dt$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[V\left(x^{(o)}_j + \gamma_j, \dots, x^{(z)}_l + \gamma_l\right) - V\left(x_j, \dots, x_l\right)\right] \mu(dv, dt) \qquad (3)$$

where: G_i - the continuous functions, γ_i - the impulse parts of right side of equations (2) ; by calculating the differentials of the second order function $\left(x^{(o)}_j\right)^p \left(x^{(s)}_k\right)^r$ and the third order function $\left(x^{(o)}_j\right)^u \left(x^{(s)}_k\right)^w \left(x^{(z)}_l\right)^y$.
The probabilistic density function of the velocities of the system's elements, which is important in the investigations of the human body response, is given by the formula:

$$p_{\dot{x}_j}(\xi_j) = \frac{1}{\sigma_{\dot{x}_j}}\left\{\varphi(\xi_j - m^{(2)}_j)/\sigma_{\dot{x}_j} - \frac{1}{6} S_{\dot{x}_j} \varphi'''(\xi_j - m^{(2)}_j)/\sigma_{x_j} + \dots\right\}$$

where skewness of density function S_{x_j} and the velocity variance $\sigma_{\dot{x}_j}$ are as follows [4]:

$$S_{\dot{x}_j} = {}^{(222)}_{jjj} - 3m^{(2)}_j \varkappa^{(22)}_{jj} + 2m^{(2)}_j \; / \; \sigma^2_{\dot{x}_j}$$

$$\sigma^2_{\dot{x}_j} = \varkappa^{(22)}_{jj} - \left(m^{(2)}_j\right)^2$$

φ is a probabilistic density function of the normal distribution and φ''' indicates $d^3\varphi \,/\, d\xi^3_i$.
The calculation of the first three moments of the $X^{(1)}(\omega, t)$ and $X^{(2)}(\omega, t)$ was reduced to solving the three systems of differential equations obtained from (2) by adapting the averaging procedure and formula [3]. These problem can be very easy in case of steady states when the influence of initial conditions can be omitted. In this case the differential equations were reduced to algebraical linear equations.

Example and discussion

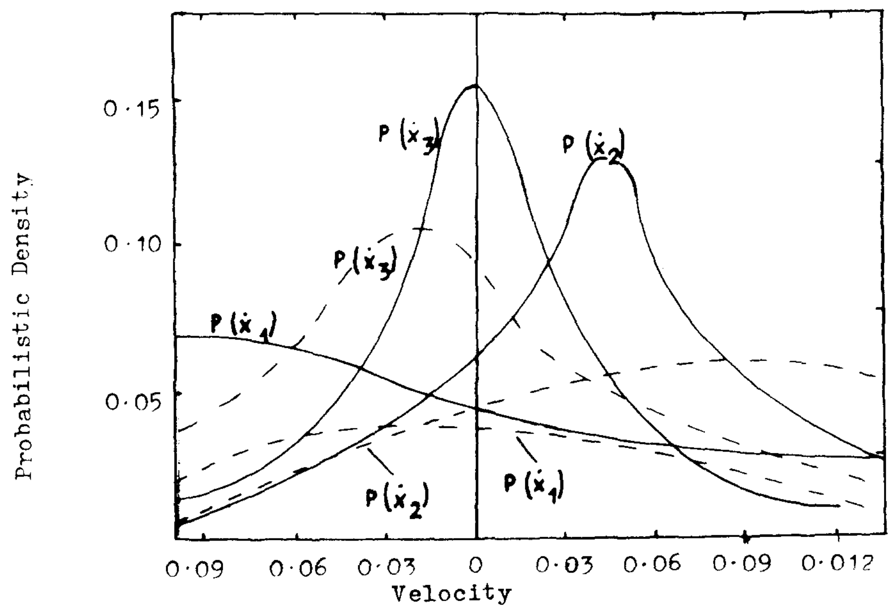
As the example let us consider the hand-arm model of the operator of a pneumatic hammer. The data for calculation were taken from [5] and were shown in Fig. 2

Fig. 2: Parameters of the hand-arm system.

	1	2	3
M kg	1·4	1·0	0·13
k $\frac{kg}{s^2}$	$4\cdot1\times10^3$	$7\cdot2\times10^4$	$2\cdot1\times10^5$
c $\frac{kg}{s}$	145	100	245

The probabilistic density functions of the velocity of the hand-arm model elements were shown in Fig.3

Fig. 3: Probability density functions of the velocity of the elements of the hand-arm system, - - - μ = 0.055 ——— μ = 0.015 .

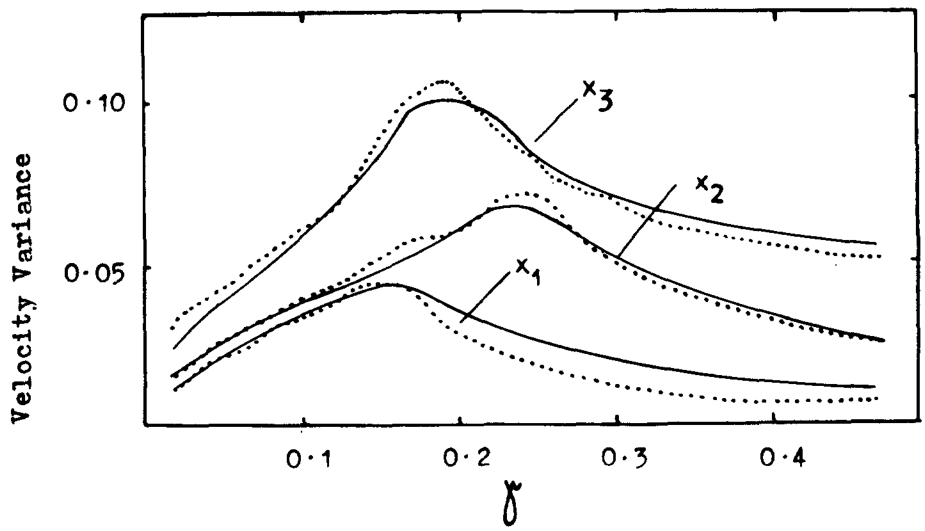


The skewness of these functions decreases for the larger values of the parameter:

$$\gamma = \lambda (1+k) \frac{m}{m+M}$$

where λ is the mean intensity of the collisions. The variance of the velocity in function of γ was presented in Fig. 4

Fig. 4: Variances of the velocity of the elements of the hand-arm system , ... experimental, ----- theoretical.

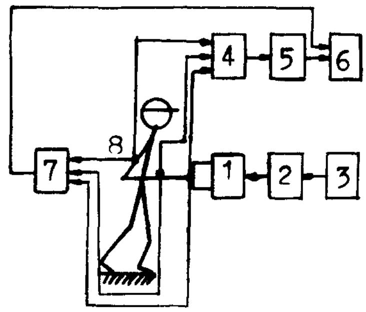


These variances increase for γ in the interval (0 , 0.25) and decrease for the larger values of γ . As the variances are the largest for the value of γ in the interval (0.15 , 0.25) which is common for many kinds of pneumatic hammers. Those vibrations can be dangerous for human being.

To verify the theoretical results, the real vibrations of the hand-arm system were investigated. The real vibrations histories of the elements of the hand-arm system were obtained during the experiment diagramaticaly shown in Fig. 5 The programming unit gave the pseudo-random impulsive excitation to the system simulating the work of pneumatic hammer.

The statistical work-up of the experimental vibration responses allowed the calculation its variances. These variances are compared with the theoretical ones in Fig. 4 Good agreement of the experimental and theoretical results was obtained. The similar experiment was done to verify the response of the oper-

Fig.5: Experimental measurment of the hand-arm system response 1 - electrodynamic vibrator, 2 - generator , 3 - programing unit, 4 - accelerometer block, 5 - integrating amplifiers, 6 - oscillograph, 7 - goniometer block, 8 - pick ups.



ator body in a standing position. Both experiments showed that the human body parameters which were identifed in case of continuous periodical excitation can be used for modelling the response to a sequence of random collisions.

The model presented assumes that only one element of the human body model is under the impulsive random excitation. However, when it is necessary to consider a model in which more than one element is excited, the same mathematical method can be easily adopted.

The model presented allows the calculation of the vibrations of the percussive machine operator's body in many possitions, i.e. standing or sitting, [7,8]. Also vibrations of a hand-arm system [5,6] and horizontal and vertical vibrations can be considered. The above mentioned model due to its simplicity can be useful for modelling the response of a various man-percussive machine systems.

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