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Controlling Chaotic Oscillators Without Feedback

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Abstract—Nonfeedback methods of controlling the chaotic behaviour of nonlinear oscillators are described. First an analytical method of controlling chaos in Duffing's oscillator which uses the classical approximate analysis of nonlinear oscillations is proposed. The second method allows us to control behaviour on the chaotic attractor in such a way as to obtain a strange nonchaotic trajectory. Finally the application of a 'dynamical absorber' to control chaos is shown. It is also shown that the presence of quasiperiodic noise in the system can limit the necessary changes of a control parameter. The methods presented are compared with the feedback Ott–Grebogi–Yorke method [E. Ott, C. Grebogi and Y. A. Yorke, *Phys. Rev. Lett.* **64**, 1196 (1990)].

1. INTRODUCTION

The presence of chaos both in nature and in man-made devices is very common and has been extensively demonstrated in the last decade. Quite frequently chaos is a beneficial feature as in some chemical or heat and mass transport problems. However, in many other situations chaos is an undesirable phenomenon leading to oscillations, irregular operations etc. Chaotic behaviour may also be detrimental to the operation of various devices as it cannot be predicted in detail.

The problem of controlling chaos, that is to convert the chaotic behaviour found in a physical system to a periodic time dependence or aperiodicity which is predictible has recently attracted interest [1–12]. The method of Ott, Grebogi and Yorke [1, 2] (OGY) has the unique feature that it enables one to select a predetermined time-periodic behaviour by making only small time-dependent perturbations. They show that permanent chaos can always be supressed by stabilizing one of many periodic orbits embedded in the chaotic attractor. The idea is to start with any initial condition, wait until the trajectory falls into a target region around the desired periodic orbit and then apply feedback control. The efficiency of this technique has been demonstrated first by Ditto *et al.* [3] in a periodically forced physical system, converting its chaotic behaviour into period-one and period-two orbits. The applications of the OGY method to stabilized higher periodic orbits in a chaotic diode resonator have been demonstrated by Hunt [4]. Another interesting application of this method is the generation of desired aperiodic orbit by Mehta and Henderson [5] and controlled transient chaos by Tel [6]. A modification of this method to control chaos using delay coordinates has been presented by Nitsche and Dressler [7]. Another related method is described by Shinbrot *et al.* [8]. It employs chaos to direct trajectories to target. Also, laminar flow has been produced in previously unstable thermal convection loops by a thermostat-type feedback mechanism in experiments by Singer *et al.* [9]. An attempt to generalize the OGY method to higher-dimensional systems is presented by Romeiras *et al.* [10].

Beside the feedback methods described above there is a possibility to stabilize periodic orbits by nonfeedback methods [11–16]. Some preliminary numerical experiments are presented in Kapitaniak [11] where stabilization of periodic orbits was achieved by adding an additional deterministic or random perturbation to a chaotic system. A nonfeedback procedure has been shown to be feasible for the periodically driven pendulum [14] and Duffing equation [15]. In this paper we discuss recently developed nonfeedback methods which are applicable to nonlinear oscillators of the Duffing type. In Section 2 we present an approximate analytical method of converting the chaotic behaviour of Duffing's oscillator into the appropriate periodic trajectory. This method is considered in noise free and noisy systems. In Section 3 we show how to control chaos to generate a strange nonchaotic trajectory. The possibility of applying a method equivalent to dynamical absorbtion to control chaos is discussed in Section 4. Finally Section 5 compares the described nonfeedback methods with the feedback OGY method.

2. ANALYTICAL METHOD OF CONTROLLING DUFFING'S EQUATION

In this section we present an analytical method of controlling the chaotic behaviour of Duffing's equation:

$$\ddot{x} + a\dot{x} + bx + cx^3 = B_0 + B_1 \cos \Omega t, \tag{1}$$

where a, b, c, B_0, B_1 and Ω are constant.

It is well known that equation (1) exhibits chaotic behaviour for certain values of the parameters [17–20]. In many cases it can be shown that chaotic behaviour is obtained through a period doubling bifurcation [18–20]. Recently, there have been some attempts to create an analytical criterion which allows us to estimate the chaotic domain in the parameter space [20, 21]. Boundaries of the chaotic zone have been obtained using the classical approximation theory of nonlinear oscillations, by examining approximate periodic solutions and studying particular types of higher order instabilities which precede the destruction of a periodic attractor in the variational Hill's type equation [22]. Now we adopt a similar procedure, the harmonic balance method, to control chaotic behaviour.

First consider the first approximate solution in the form:

$$x(t) = C_0 + C_1 \cos(\Omega t + \psi), \tag{2}$$

where C_0, C_1 and ψ are constants. Substituting equation (2) into equation (1) it is possible to determine these constants [20, 22]. To study the stability of the solution (2) a small variational term $\delta x(t)$ is added to equation (2) as:

$$x(t) = C_0 + C_1 \cos(\Omega t + \psi) + \delta x(t). \tag{3}$$

After some algebraic manipulations, the linearized equation with periodic coefficients for $\delta x(t)$ is obtained

$$\delta\ddot{x} + a\delta\dot{x} + \delta x[\lambda_0 + \lambda_1 \cos \Xi + \lambda_2 \cos 2\Xi] = 0, \tag{4}$$

where: $\lambda_0 = 3C_0^2 + (3/2)C_1^2$, $\lambda_1 = 6C_0C_1$, $\lambda_2 = (3/2)C_1^2$, $\Xi = \Omega t + \psi$. In the derivation of equation (4), for simplicity, it was assumed without loss of generality that $b = 0$. As we have a parametric term of frequency $\Omega - \lambda_1 \cos \Xi$, the lowest order unstable region is that which occurs close to $\Omega/2 \simeq \sqrt{\lambda_0}$ and at its boundary we have the solution:

$$\delta x = b_{1/2} \cos((\Omega/2)t + \psi). \tag{5}$$

To determine the boundaries of the unstable region we insert equation (5) into equation (4), and the conditions of nonzero solution for $b_{1/2}$ lead us to the following criterion to be satisfied at the boundary:

$$(\lambda_0 - \Omega^2/4)^2 + a^2\Omega^2/4 - \lambda_1^2/4 = 0. \tag{6}$$

From equation (6) one obtains the interval $(\Omega_1^{(2)}, \Omega_2^{(2)})$ within which period-two solutions exist. Further analysis shows that at Ω_2 we have a stable period-doubling bifurcation for decreasing Ω and at Ω_1 an unstable period-doubling bifurcation for increasing Ω [20]. In this interval we can consider the period-two solution of the form:

$$x(t) = A_0 + A_{1/2}\cos((\Omega/2)t + \eta) + A_1\cos\Omega t, \tag{7}$$

where A_0, $A_{1/2}$, A_1 and η are constants to be determined. Again to study the stability of the period-two solution we have to consider a small variational term $\delta x(t)$ added to equation (7). The linearized equation for $\delta x(t)$ has the following form:

$$\delta\ddot{x} + a\delta\dot{x} + \delta x[\lambda_0^{(2)} + \lambda_{1/2c}\cos(\Omega/2)t + \lambda_{1/2s}\sin(\Omega/2)t + \lambda_{3/2}\cos((3\Omega/2)t + \eta) + \lambda_{1c}^{(2)}\cos\Omega t + \lambda_{1s}^{(2)}\sin\Omega t + \lambda_2^{(2)}\cos 2\Omega t] = 0, \tag{8}$$

where:

$\lambda_0^{(2)} = 3(A_0^2 + 0.5A_{1/2}^2 + 0.5A_1^2)$, $\lambda_{1/2c} = 3A_{1/2}(2A_0 + A_1)\cos\eta$,
$\lambda_{1/2s} = 3A_{1/2}(A_1 - 2A_0)\sin\eta$, $\lambda_{3/2} = 3A_1A_{1/2}$, $\lambda_{1c}^{(2)} = 6A_0A_1 + (3/2)A_{1/2c}^2\cos 2\eta$,
$\lambda_{1s}^{(2)} = -(3/2)A_{1/2s}^2\sin 2\eta$, $\lambda_2^{(2)} = (3/2)A_1^2$.

The form of equation (8) enables us to find the range of existence of a period-four solution, represented by:

$$\delta x = b_{1/4}\cos((\Omega/4)t + \eta) + b_{3/4}\cos((3\Omega/4)t + \eta). \tag{9}$$

After inserting equation (9) into equation (8) the condition of having nonzero solutions for $b_{1/4}$ and $b_{3/4}$ gives us the following set of nonlinear algebraic equations for Ω, $\cos\eta$ and $\sin\eta$ to be satisfied for existence:

$$\begin{aligned}
&(\lambda_{1/2s} + \lambda_{1s}^{(2)}) - 0.5(\lambda_{1/2c} - \lambda_{1c}^{(2)})(-a\Omega/2 + \lambda_{1/2s} - \lambda_{3/2}\sin\eta) = 0,\\
&((9/8)\Omega^2 + 0.5\lambda_0 + \lambda_{3/2}\cos\eta) - 0.5(\lambda_{1/2c} + \lambda_{1c}^{(2)})(\lambda_{1s}^{(2)} + \lambda_{1/2c}) = 0,\\
&(-(3/2)a\Omega - \lambda_{3/2}\sin\eta) - 0.5(\lambda_{1/2c} + \lambda_{1c}^{(2)})(\lambda_{1s}^{(2)} + \lambda_{1/2c}) = 0.
\end{aligned} \tag{10}$$

Solving equation (10) by a numerical procedure it is possible to obtain $\Omega_1^{(4)}$ and $\Omega_2^{(4)}$, the frequencies of stable and unstable period-four bifurcations.

Now we assume that the Feigenbaum model [23] of period doubling is valid for our system i.e.:

$$\left.\frac{\Omega_{1,2}^{2^n} - \Omega_{1,2}^{2^{n-1}}}{\Omega_{1,2}^{2^{n+1}} - \Omega_{1,2}^{2^n}}\right|_{n\to\infty} \to \delta \tag{11}$$

where $\delta = 4.669\ldots$ is the universal Feigenbaum constant and $n = 1, 2, \ldots$.

It has been proved that period-doubling bifurcations in one-dimensional maps with a single hump fulfil the Feigenbaum model, but there are a number of examples where this model can be taken as good approximation to the real phenomena in higher-dimensional dissipative systems [24, 25].

To obtain approximate values of the limits of period-doubling bifurcations (accumulation points) we replaced the limit in equation (11) by an equality. This allows us to indicate

$$\Delta\Omega_{1,2}^{2^n} = \Omega_{1,2}^{2^n} - \Omega_{1,2}^{2^{n-1}}. \tag{12}$$

Now it is easy to show that $\Delta\Omega_{1,2}^{2^n}$, $\Delta\Omega_{1,2}^{2^{n-1}}$, ... forms an infinite geometrical series with a ratio $1/\delta$. With both stable and unstable period-two and period-four boundaries using the approximation described above one obtains:

$$\Omega_1^{(\infty)} = \Omega_1^{(2)} + \Delta\Omega_1/(1 - 1/\delta),$$
$$\Omega_2^{(\infty)} = \Omega_2^{(2)} - \Delta\Omega_2/(1 - 1/\delta),$$

where: $\Delta\Omega_1 = \Omega_1^{(4)} - \Omega_1^{(2)}$, $\Delta\Omega_2 = \Omega_2^{(2)} - \Omega_2^{(4)}$.

The domain where chaotic behaviour can occur is proposed to lie between the limits of unstable and stable period-doubling cascades, in the interval $(\Omega_1^{(\infty)}, \Omega_2^{(\infty)})$ and of course to expect chaos one must have:

$$\Omega_1^{(\infty)} < \Omega_2^{(\infty)}.$$

More details about this method can be found in [11, 21]. Beside estimating the chaotic region in parameter space this approach evaluates analytically the approximate unstable orbits or at least regions in parameter space where they exist. The above analysis can be used to control equation (1) by changing parameter Ω. It should be noted here that the frequency Ω is the parameter which can be easily changed in real experimental systems modelled by equation (1). From Fig. 1 one can find that changing Ω by $\Omega^* < 0.12$ one can obtain different types of periodic behaviour; from period-one to theoretically period-2^n ($n = 1, 2, \ldots$). Periodic orbits of higher order ($n > 4$) are difficult to obtain as the Ω intervals of the existence of these solutions are very small. Examples of controlling a few of the periodic orbits are shown in Fig. 2. We plot coordinate x of the Poincare map as a

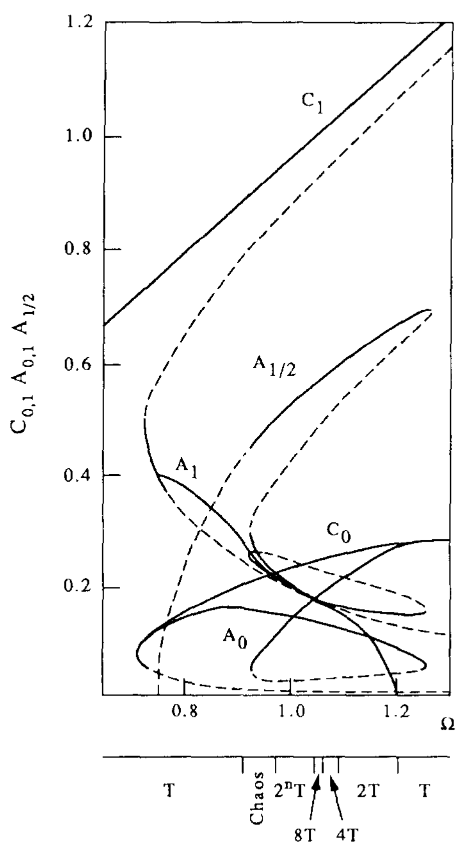

Fig. 1. Parameters of period-one and period-two approximate solutions (2) and (7), and the W intervals of existence of period-four and period-eight solutions; $a = 0.05$, $b = 0$, $c = 1$, $B_0 = 0.03$, $B_1 = 0.16$. The solid line indicates stable solutions while the broken line indicates unstable solutions.

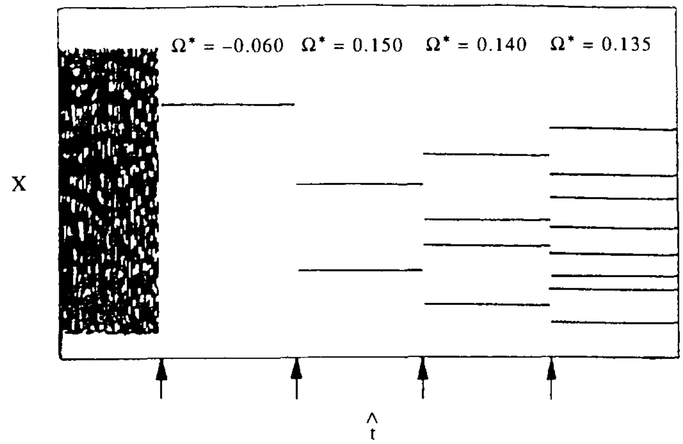

Fig. 2. Successive control of period-one, -two, -four, -eight orbits. The arrows indicate the times of switching; $a = 0.05$, $b = 0$, $c = 1$, $B_0 = 0.03$, $B_1 = 0.16$, $\Omega = 0.97$.

function of discrete time $\hat{t} = 2\pi n/(\Omega + \Omega^*)$, $n = 1, 2, \ldots$ The frequency perturbations were programmed to successively control four different periodic orbits. The times at which we switched the control from stabilizing one periodic orbit to stabilize another are labelled by arrows in Fig. 2.

Recently it has been shown that the small additive quasiperiodic noise of the form:

$$h(t) = \sum_{i=1}^{N} A_i \cos(v_i t + \phi_i), \tag{13}$$

where $A_i \ll B_{0,1}$ are constants, v_i and ϕ_i are time independent random variables, shifts the period doubling bifurcation point decreasing the zone of existence of each period-2^n solution and can even eliminate chaos [11, 16].

This phenomenon shows that we can reduce the range in which the control parameter is allowed to vary by simultaneously adding noise into the system i.e.

$$\ddot{x} + a\dot{x} + bx + cx^3 = B_0 + B_1 \cos((\Omega + \Omega^*)t) + \sum_{i=1}^{N} A_i \cos(v_i t + \phi_i). \tag{14}$$

Quasiperiodic noise given by equation (13) is an approximation of the realization of the band-limited white noise stochastic process with zero mean and a spectral density:

$$s(v) = \begin{cases} s/(v_{\max} - v_{\min}) & v \in [v_{\min}, v_{\max}] \\ 0 & v \notin [v_{\min}, v_{\max}], \end{cases}$$

where s is the intensity of noise and $[v_{\min}, v_{\max}]$ is the interval of considered frequencies [11, 16] and can be easily simulated experimentally.

An example of this type of control is shown in Fig. 3. In this case as an effect of control we obtain solutions of appropriate period perturbed by noise (13).

3. GENERATION OF STRANGE NONCHAOTIC TRAJECTORY

A nonfeedback technique which allows us to generate a strange nonchaotic trajectory by a small change of system parameters is described in this section. Our method is applicable to the systems whose behaviour depends on a control parameter c in such a way that they

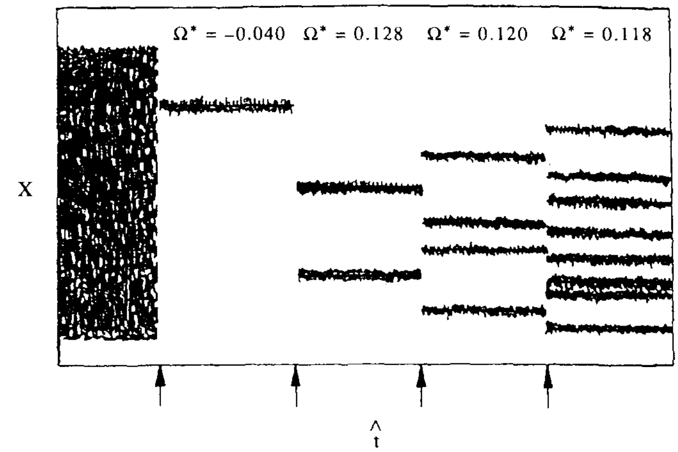

Fig. 3. Successive control of period-one, -two, -four, -eight orbits with noise added to the system. The arrows indicate the times of switching; $a = 0.05$, $b = 0$, $c = 1$, $B_0 = 0.03$, $B_1 = 0.16$, $\Omega = 0.97$, $A_i = 0.004$, $v_{min} = 0.9$, $v_{max} = 1.1$, $N = 200$.

have a chaotic attractor for one value of c, say c_1 and a strange repeller together with a periodic attractor for the other value of $c - c_2$. Systems with a large strange repeller exhibit transient chaos [26, 27]. Trajectories started from randomly chosen initial points then approach the attractor with probability one. Before reaching it, however, they might come close to the strange repeller and stay in its vicinity for a shorter or longer time. Long lived chaotic transients are often present around crisis configurations [26], at parameter values just beyond the disappearance of the chaotic attractor. It is worth mentioning that systems with fractal basin boundaries [28] are also accompanied by transient chaos since such boundaries are, in general the stable manifolds of a chaotic repeller. Computation of transient Lyapunov exponents for the systems with chaotic repellers often shows such a property that their values become nonpositive far before the transient died i.e. before trajectory reaches the attractor [29, 30].

The main idea of our method is described in Fig. 4. Let's consider two trajectories which start from nearby initial conditions A' and A'' which lie on or close to a strange chaotic attractor. For $t_0 < t < t_1$ these trajectories represent evolution of the system for a value of control parameter $c = c_1$ and for $t_1 < t < t_3$ the evolution is shown for control parameter

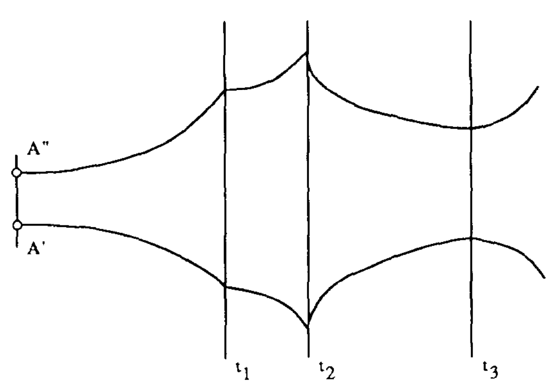

Fig. 4. Behaviour of nearby trajectories.

$c = c_2$. In the first time interval we observe exponential divergence of trajectories described by a positive Lyapunov exponent—$\lambda(t) > 0$. At time $t = t_1$ we are changing the value of the control parameter from c_1 to c_2. For a new value of control parameter our system has a strange repeller and we observe first a further divergence of trajectories and a positive transient Lyapunov exponent $\lambda(t) > 0$ for $t_1 < t < t_2$. At time t_2 the transient Lyapunov exponent changes sign from positive to negative and for $t_2 < t < t_3$ we observe a convergence of trajectories. At time t_3 we are changing the value of the control parameter to c_1 again, etc. If t_3 is chosen in such a way that the period of time $t_3 - t_2$ is not sufficient for a system to reach a periodic attractor and

$$\int_{t_0}^{t_2} \lambda(t)\,\mathrm{d}t \approx \int_{t_2}^{t_3} \lambda(t)\,\mathrm{d}t, \tag{15}$$

then we do not observe divergence of trajectories in time. As a part of a trajectory evolves on the strange chaotic attractor the switches between c_1 and c_2 will take place in different points of phase space so the trajectory is aperiodic (not t_3-periodic).

As an example of our method let us consider:

$$\ddot{x} + a\dot{x} - (1 + c\cos\omega t)x + bx^3 = 0, \tag{16}$$

where a, b, c and ω are constant. Examples of this equation are found in many applications of mechanics, particularly in problems of the dynamic stability of elastic systems [31, 32].

Equation (16) has three Lyapunov exponents: one of them is always zero, one is always negative and the third one can change its sign with the change of control parameter c. The value of this Lyapunov exponent is responsible for exponential divergence or convergence of nearby trajectories and in the rest of this paper we will investigate only the value of this Lyapunov exponent—λ. If λ is negative we have the limit cycle attractor, if λ is positive we have a strange chaotic attractor and if $\lambda = 0$ a torus is an attractor.

The behaviour of equation (2) has been investigated numerically in [30], where equation (2) has been integrated by the fourth-order Runge–Kutta method with the integration step being $T_1/200$, where $T_1 = 2\pi/\omega$ and Lyapunov exponents have been obtained using the method of Wolf *et al.* [33]. The plot of λ vs control parameter c is shown in Fig. 5 as c changes from 0 to 0.5. If c is small, λ is negative, so equation (2) does not show sensitive dependence on the initial conditions. When c is increasing up to about 0.348, λ changes suddenly from negative to positive values and the behaviour of equation (2) is chaotic. In the interval $c \in (0.256, 0.348)$ we observe long transient aperiodic trajectories without sensitive dependence on the initial condition i.e. the transient Lyapunov exponent $\lambda(t)$ decreases and turns negative far before the transients have died. The lifetime of the observed transient is relatively long (10–10 T_1) in comparison to these times. The lifetime of transient chaos (when transient Lyapunov exponent is positive) is much shorter (about 10 T_1).

In our control procedure we took one value of c for which the behaviour of the system is chaotic and one value of c for which equation (2) has the above mentioned transient behaviour. In numerical investigations $c_1 = 0.35$ and $c_2 = 0.34$ have been taken and we consider the behaviour of the system:

$$\ddot{x} + a\dot{x} - (1 + c(t)\cos\omega t)x + bx^3 = 0, \tag{17}$$

where $c(t) = c_1$ for $t_0 < t < t_1$ and $c(t) = c_3$ for $t_1 < t < t_3$. Generally T_1 and $T_2 = t_3$ are incommensurate so equation (3) has four-dimensional phase-space (x, $\dot{x}$, ωt, $(2\pi/t_3)t$) and a strange nonchaotic attractor may occur. In Fig. 6 we showed the plot of transient Lyapunov exponent $\lambda(t)$ vs t. As a function $c(t)$ is discontinuous in numerical calculation

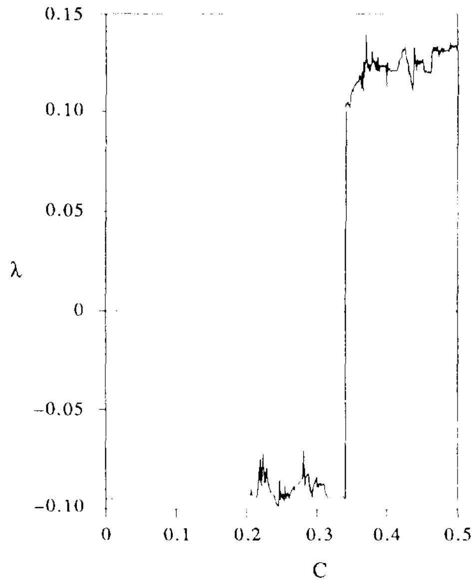

Fig. 5. Lyapunov exponent λ for equation (2) vs control parameter c.

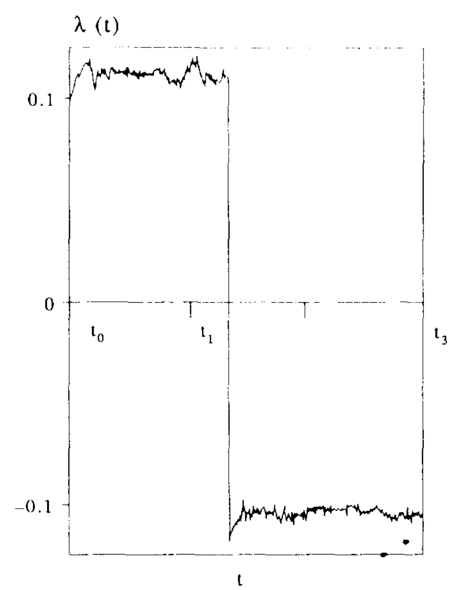

Fig. 6. Transient Lyapunov exponent $\lambda(t)$ for equation (3) vs time.

we took its Fourier series approximation with 100 components. In this figure the regions of divergence and convergence of nearby trajectories are visible. As the value of Lyapunov exponent averaged over time T_2 is negative and close to -0.001 we do not observe exponential divergence of trajectories in time. This result shows that applying our control procedure we manage to build an aperiodic trajectory which is predictable in the sense that nearby trajectories do not diverge exponentially, this is shown in Fig. 7. Figure 7 presents

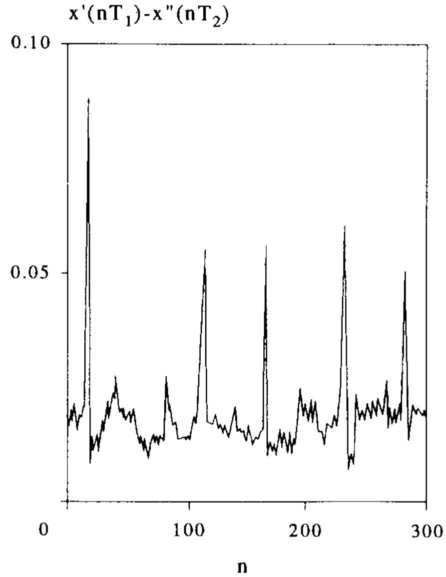

Fig. 7. Distance between nearby trajectories of equation (3) vs time.

the distance: $x'(t + nT_2) - x''(t + nT_2)$; where $n = 1, 2, \ldots, x'$ is a trajectory for initial conditions $x(0) = 0.1$, $\dot{x}(0) = 0$ and x'' is a trajectory for $x(0) = 0.101$, $\dot{x}(0) = 0$.

In this method to obtain a desired aperiodic orbit we took advantage of the fact that generally the addition of new excitations with incommensurate frequencies drives a dynamical system closer and closer to the ergodic state. This phenomenon has been observed in a number of numerical experiments [11, 34, 35] and recently El Naschie explained it analytically using the properties of multi-dimensional Cantor sets [36, 37].

4. CONTROL VIA DYNAMICAL ABSORBER

In the methods described in previous sections the appropriate control has been achieved by adding perturbation into a chaotic system or by small change of one of the parameters. In this section we describe a different method in which the control effect is obtained by joining the chaotic system with another system. The idea of this method is similar to the work of the so-called dynamical vibration absorber. A dynamical vibration absorber has one degree of freedom, usually a mass on the spring, which is added to the main system (see Fig. 8) to shift a resonance zone. The parameters of the absorber are small in comparison to the parameters of the main system so they can be easily added to the existing systems without change to its construction.

To explain the role of dynamical absorbers in the procedure of controlling chaotic behaviour let us again consider Duffing's oscillator (1), this time coupled with an additional linear system:

$$\begin{aligned} &\ddot{x} + a\dot{x} + bx + cx^3 + d(x - y) = B\cos\Omega t, \\ &y + e(y - x) = 0, \end{aligned} \tag{18}$$

where a, b, c, d, e, B and Ω are constant. The constant e is the characteristic parameter for the absorber and we take it here as a control parameter. Parameters of equation (18)

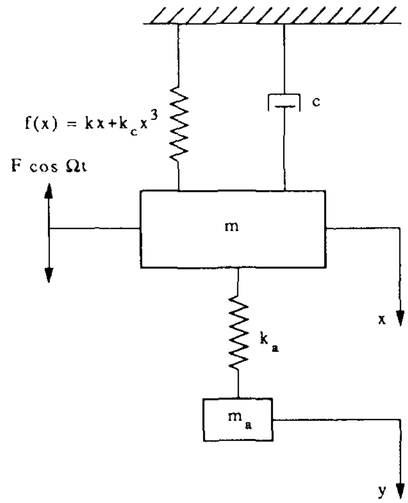

Fig. 8. The dynamical absorber.

are related to those of Fig. 8 in the following way: $a = c/m\Omega$, $b = k/m\Omega^2$, $c = k_c/m\Omega^2$, $d = k_a/m\Omega^2$, $e = k_a/m_a\Omega^2$, $B = F/m\Omega^2$.

The controlling effect of a dynamical absorber can be seen in Fig. 9 where we show a plot describing a different periodic behaviour vs the absorber characteristic parameter e.

5. CONCLUSIONS

Nonfeedback methods of controlling chaos presented in this paper can be easily used for controlling the dynamical behaviour of nonlinear oscillators.

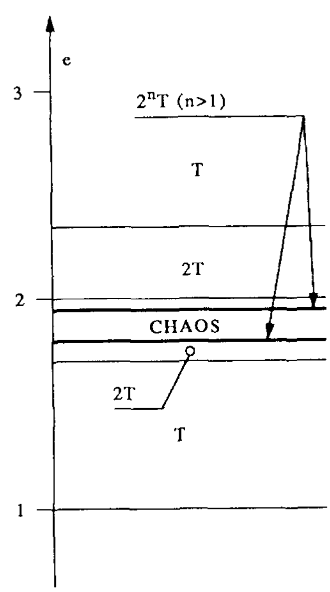

Fig. 9. Controlling effect of dynamical absorber; $a = 0.168$, $b = 0.5$, $c = 0.5$, $B = 0.21$, $\Omega = 1$.

The analytical technique of controlling chaos in Duffing's equation presented in Section 1 is based on:

(a) the approximate period-one, -two and -four solutions and their stability limits computed by the harmonic-balance method,
(b) Feigenbaum's universal constant for the asymptotic ratio of the stability intervals of the 2^n and 2^{n+1} period solutions.

It can be applied to the class of nonlinear oscillators which the harmonic-balance method analysis shows the possibility of period-doubling bifurcation ($\lambda_1 \neq 0$ in equation (4)). Generally this is, of course, a very small category of problems, but they have a great number of applications.

The method described in Section 3 shows how to convert a chaotic trajectory into an aperiodic trajectory which does not show sensitive dependence on initial conditions. This control although it does not change a great deal the features of a single trajectory (for example phase plot, power spectum) it does improve predictibility.

An interesting method of controlling chaos is the application of an additional system which connected with a main system can convert its behaviour from a chaotic to a periodic one. This method can be easily applied to the experimental systems, especially mechanical ones, where the similar method (dynamical absorber) is well known as a way to shift a resonance zone.

The methods presented are less general than the feedback method of Ott–Grebogi–Yorke. They can be applied only to systems which displayed specific a priori known behaviour. This requires the knowledge of equations of motion. On the other hand, to apply this method we do not have to follow the trajectory. The control procedures can be applied at any time and we can switch from one periodic orbit to another without returning back to the chaotic behaviour, although after each switch transient chaos is observed. The life time τ of this transient chaos strongly depends on initial conditions and for example we observed $\tau < 400(\Pi/(\Omega + \Omega^*)$ for a method described in Section 1. In a nonfeedback method we do not have to wait until the trajectory is close to an appropriate unstable orbit. In some cases this time can be quite long.

An interesting feature of nonfeedback methods is the possibility of using additional noise to reduce the necessary range of control parameter variation. This is important as our method requires larger perturbations of the control parameter than the method of [1, 2].

Generally, nonfeedback methods can be useful tools of controlling chaos especially in man designed devices where the dynamics are known despite the fact that at first sight they seem to be based on the 'brutal' change of the system parameters or even configuration. Until now the main reason for investigating chaotic behaviour in practical (mainly engineering) systems was to estimate the zone in parameter space where chaos can be found. This estimation was necessary to build a real system so that it is operating as far as possible from the chaotic zone. Nonfeedback methods described in this paper allow another approach to build a system as a chaotic one and to obtain an appropriate response by controlling it. The application of the described methods does not require a series of changes in the system and allow for easy transition from one type of response to another.

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