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LETTERS TO THE EDITOR

INTERPRETATION OF APERIODIC TIME SERIES: A NEW VIEW OF DRY FRICTION

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1. INTRODUCTION

The objective of the investigations reported was two-fold: firstly, to throw light on the rather poorly understood relationship between frictional force and relative velocity in dry sliding contact between solids; and secondly, to generate data from a real mechanical non-linear system forced quasiperiodically, and to use methods of analysis of aperiodic time series to look for regions of existence of chaotic and non-chaotic strange attractors for the system.

Earlier work on quasiperiodically forced oscillators [1–4] had led us to expect robust existence of the latter in some regions of parameter space. The results were somewhat unexpected, and led us to take a novel view of the importance to the behaviour of the friction force of motion normal to the mean plane of contact.

In section 2 we discuss the phenomenon of dry friction, describe the experimental configuration and present some experimental results. In section 3 we discuss the analysis of the experimental data and, finally, in section 4 we interpret these results in terms of a model for the observed chaotic behaviour.

2. EXPERIMENTS

The friction properties of sliding bodies are important in a large number of engineering applications. However, no universal mathematical model has yet been found which satisfactorily describes this physical phenomenon. The nature of the dynamic friction forces developed between objects in contact is extremely complicated and is affected by many factors such as the frequency of the contact, the response of the interface to normal forces, the roughness of the surfaces, the wear, the lubrication, the type of interface and others. Dynamic friction is not a simple phenomenon but comprises a set of factors different in nature and behaviour, which cannot be fully described by a simple analytical equation.

Most experimental studies [5–9] show that a dynamical friction force acts in the same direction as the relative velocity of the bodies in contact but in the opposite sense, generally according to the relationship:

$$F \sim \mathrm{sign}\,(V_r), \tag{1}$$

where F represents the friction force and V_r the relative velocity.

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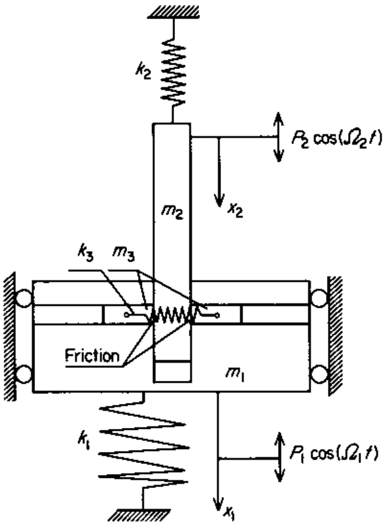

Figure 1. The experimental rig.

The friction force has been found to be an irreversible function of the sliding velocity in all cases in that the accelerating and decelerating branches of the friction–velocity curves are distinct in a cycle of motion. For different pairs of metals in contact, the same lubrication conditions may produce curves very distinct in shape, and even for the same combination of metals the shape, slope and separation between the two branches is very much dependent on both the dynamic properties of the test rig and the driving velocity. As a result the experimental characteristics are not defined uniquely by the nature of contacting bodies, but are functions of all the dynamical properties of the system.

Experimental measurements of the dry friction force between steel and brass in contact are presented, as obtained by using the equipment shown in Figure 1. The test rig provides additional facilities for dry friction tests compared with those used in previous experiments, where in the latter the velocity of one of the contacting bodies was constant. In this test rig, all the bodies in contact are oscillating. Two simple subsystems, each with its own driving force, $P_i \cos \Omega_i t$, are coupled through the frictional contact between brass blocks placed in the bottom subsystem and held by springs in permanent contact with the steel shaft of the top subsystem. Friction forces were measured by using a piezo-electric force transducer connected to a charge amplifier and a 12-bit data acquisition system in a microcomputer. Simultaneously, other system variables, for example velocities and displacements of both subsystems, were measured.

In addition to the forced vertical oscillations of the masses 1 and 2, it was found that the masses 3 oscillate horizontally. These horizontal oscillations are caused by the roughness of the sliding surfaces of the masses 1 and 3. It is well known that even the most polished metallic surfaces are not perfectly flat. Under magnification, one observes that these surfaces have undulations that form hills and valleys the dimensions of which are large in comparison with molecular dimensions (see Figure 2). This horizontal degree of freedom, although very small, nevertheless appears to impose its signature on the overall dynamics of the system.

Figure 2. A typical profile of mild steel specimen after surface grinding and polishing.

A full description and discussion of the experimental technique and results will appear elsewhere [10, 11]. Here, in Figures 3(a–c), we reproduce some typical examples of friction force *vs.* relative velocity dependence in a quasiperiodically excited two-degree-of-freedom system. In Figure 3(a) the dependence is approximately described by equation (1). This case was obtained for values of Ω_1 and Ω_2 which were close together ($\Omega_1 = 10{\cdot}51$ Hz, $\Omega_2 = 14{\cdot}61$ Hz). In Figures 3(b) and 3(c), cases in which the value Ω_2 is substantially larger than Ω_1 ($\Omega_1 = 10{\cdot}51$ Hz, with $\Omega_2 = 30{\cdot}83$ Hz in Figure 3(b) and 35·03 Hz in Figure 3(c)), the friction–velocity relation seems to be completely different from that of equation (1); force–velocity changes are now highly unpredictable and complicated.

3. ANALYSIS AND INTERPRETATION OF THE DATA

The behaviour displayed in Figures 3(b) and (c) is clearly aperiodic, and a first objective of analysis is to determine the character of the attractor and behaviour of trajectories near it. Many dissipative dynamical systems exhibit strange behaviour, and two classes of strange attractor seem to be characteristic of quasiperiodically forced systems [1–4, 12, 13]: (a) a strange chaotic attractor—one which is geometrically, "strange", i.e., the attractor is neither a finite set of points nor is it piecewise differentiable, and for which typical orbits have positive Lyapunov exponents, implying exponential divergence with time of nearby orbits; (b) a strange non-chaotic attractor—one which is also geometrically "strange" and has a fractal dimension like a typical chaotic attractor, but for which typical nearby orbits do not diverge exponentially with time.

Since both types of strange attractor are geometrically similar, some qualitative measure, such as the value of Lyapunov exponents and of the information dimension connected with them, is necessary to distinguish these classes.

In earlier work [12, 13] numerical experiments were employed to show that it is impossible to distinguish between strange chaotic and non-chaotic attractors on the basis of Lyapunov exponents estimated from time series. However, it was indicated [13, 14] that the information dimension of the attractor might be used to make the distinction.

In what follows we develop this idea further by using the results of section 2 which show complicated aperiodic behaviour. Our approach follows that of Grassberger and Procaccia [15]. We constructed an m-component "state" vector X_i from a time series of the friction force $F(t)$ as

$$X_i = \{F_1(t_1), F_2(t_1+\tau), \ldots, F_m(t_1+(m-1)\tau\},$$

where τ is an appropriate time delay (of the order of characteristic physical time scales) and used the correlation integral defined for N vectors distributed in an m-dimensional space as a function of the distance r:

$$C(r, m) = \lim_{N\to\infty} (1/N^2) \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta(r - |x_i - x_j|),$$

where Θ is the Heaviside step function. If the number of points is large enough, as assumed above, this distribution will obey a power-law scaling with r for small r, $C(r, m) \sim r^{\nu}$, where ν is the correlation dimension. As we increase m, the correlation dimension is seen to converge to its true value.

The results of this procedure applied to the data of Figure 3(b) are shown in Figure 4. We find that the correlation dimension converges to a value greater than six for the embedding dimension $m = 8$, and appears not to change for further increase of m. The data of Figure 3(c) give results which are very similar: the value of ν at $m = 8$ differs by only 0·1.

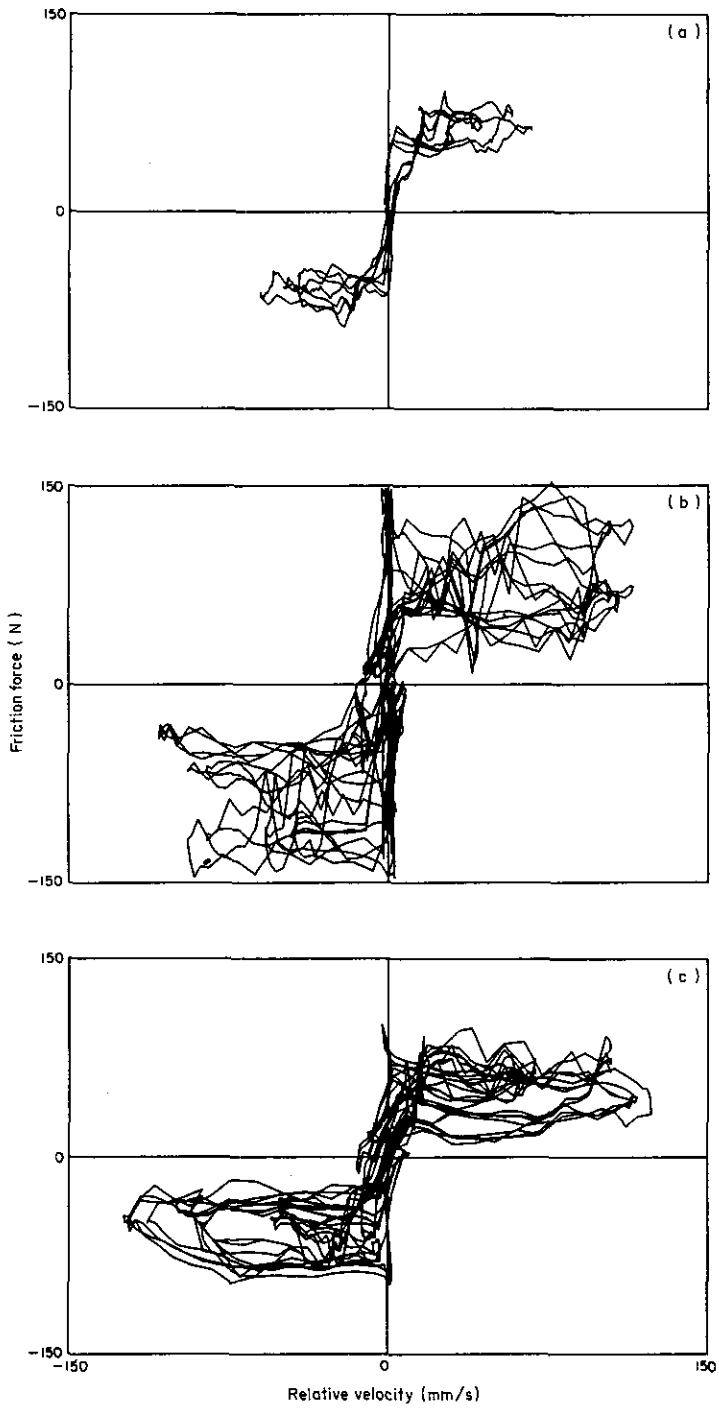

Figure 3. Examples of friction force–relative velocity diagrams. (a) $\Omega_1 = 10{\cdot}51$ Hz, $\Omega_2 = 14{\cdot}01$ Hz; (b) $\Omega_1 = 10{\cdot}51$ Hz, $\Omega_2 = 30{\cdot}83$ Hz; (c) $\Omega_1 = 10{\cdot}51$ Hz, $\Omega_2 = 35{\cdot}03$ Hz.

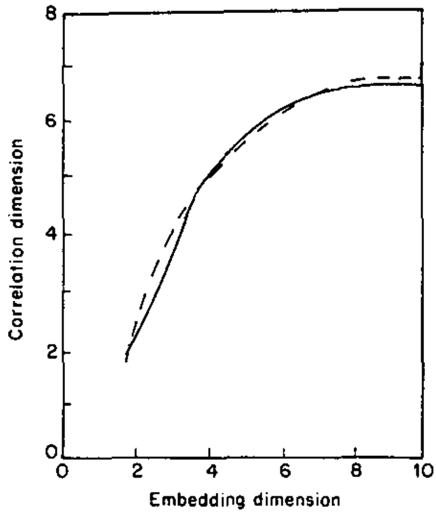

Figure 4. Correlation dimension *vs.* embedding dimension plot. ——, Friction force time series of Figure 2(b); – – – –, friction force time series of Figure 2(c).

4. A MODEL FOR THE OBSERVED BEHAVIOUR

The equations for the vertical motion of the two oscillators take the form

$$(m_3+m_1)\ddot{x}_1+f(x_i,\dot{x}_i,N)+k_1x_1=P_1\cos\Omega_1t, \qquad m_2\ddot{x}_2+f(x_i,\dot{x}_i,N)+k_2x_2=P_2\cos\Omega_2t, \tag{2}$$

where N is the total normal force acting at the sliding contacts.

If we assume N to be constant, flow defined by his equation occupies a six-dimensional phase space and can be described by six one-dimensional Lyapunov exponents. Two of them, connected with the forcing, are explicitly zero. Of the other four, at least two must be negative to reflect the dissipative nature of the system. Hence the Lyapunov dimension, given by

$$d_L=j+\sum_{i=1}^{j}\lambda_i/|\lambda_{j+1}|, \tag{3}$$

where j is the largest number of Lyapunov exponents for which the sum $\sum_{i=1}^{j}\lambda_i$ is non-negative, is at most four if there are no positive Lyapunov exponents. A value of d_L greater than four indicates that at least one Lyapunov exponent must be positive and that the system is chaotic.

Kaplan and Yorke [16] have suggested the equality of the Lyapunov dimension and the information dimension, d_i, and it has been shown [15] that the correlation dimension ν and information dimension d_i satisfy

$$\nu\leqslant d_i. \tag{4}$$

Hence the experimental data suggest that, since $\nu>6$ and hence $d_i>6$, the model represented by equations (2) is inadequate to describe the dynamics, since its phase space dimension is too low to produce such a value for d_i.

We propose that the reason for the inadequacy is the assumption in equations (2) of constant N. A more adequate model must allow for variations in N associated with small irregularities in the surfaces in contact and motion normal to the plane of contact. We propose that N can vary and is connected to the system (2) through an equation of the form

$$m_3\ddot{y}_3 + k_3 y_3 - N = \text{constant}, \tag{5}$$

governing the dynamics of this normal motion, where $y_3(x_1, x_2)$ is the local departure of the separation of the brass slides from its mean value as a result of surface roughness.

This equation, added to equations (3), produces an eight-dimensional system. An embedding dimension of eight appears to give a good approximation for the value of v and suggests that consideration of the degree of freedom normal to the direction of slipping is important in modelling the friction force. Calculations of v for other values of Ω_1 and Ω_2 support the converged value closed to 6·70 for an embedding dimension of eight.

Finally, equations (2) and (5) have eight one-dimensional Lyapunov exponents. The greatest possible Lyapunov dimension for the case in which there is no positive Lyapunov exponent is six. Hence our experimental data suggest that the behaviour of the system is chaotic.

5. SUMMARY

(1) The behaviour of this system in which two linear oscillators are connected through a dry friction force is undoubtedly chaotic, as the value of $d_i > 6$.

(2) The inclusion of a degree of freedom normal to the plane of slipping is important in modelling the experimental consequences of dry friction.

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