Express Letters

Experimental Hyperchaos in Coupled Chua's Circuits

Tomasz Kapitaniak, Leon O. Chua, and Guo-Qun Zhong

***Abstract*—In this letter we report experimental observation of hyperchaotic attractors in open and closed chains of Chua's circuits.**

I. Introduction

In the last 20 years it has been shown that chaotic behavior is typical for three dimensional systems [1]–[3]. In higher (at least four) dimensional systems, besides chaotic attractors characterized by one positive Lyapunov exponent, it is possible to find hyperchaotic attractors with two (or more) positive Lyapunov exponents. Hyperchaotic attractors have been observed in a number of numerical studies [4]–[8]. Laboratory experiments have also revealed the existence of hyperchaos in hydrodynamic systems [20] and semiconductor device [21].

II. Hyperchaotic Attractors

In what follows we investigate the hyperchaotic attractors in a chain of coupled identical Chua's circuits, as shown in Fig. 1. The state equations for the circuit of Fig. 1 are as follows

$$
\begin{aligned}
C_1\frac{dv_{C_1}^{(1)}}{dt} &= G\left(v_{C_2}^{(1)} - v_{C_1}^{(1)}\right) - f\left(v_{C_1}^{(1)}\right)\\
C_2\frac{dv_{C_2}^{(1)}}{dt} &= G\left(v_{C_1}^{(1)} - v_{C_2}^{(1)}\right) + i_L^{(1)} + K_1\left(v_{C_2}^{(2)} - v_{C_2}^{(1)}\right)\\
L\frac{di_L^{(1)}}{dt} &= -v_{C_2}^{(1)}\\
C_1\frac{dv_{C_1}^{(2)}}{dt} &= G\left(v_{C_2}^{(2)} - v_{C_1}^{(2)}\right) - f\left(v_{C_1}^{(2)}\right)\\
C_2\frac{dv_{C_2}^{(2)}}{dt} &= G\left(v_{C_1}^{(2)} - v_{C_2}^{(2)}\right) + i_L^{(2)} + K_2\left(v_{C_2}^{(3)} - v_{C_2}^{(2)}\right)\\
L\frac{di_L^{(2)}}{dt} &= -v_{C_2}^{(2)} \qquad\qquad (1)\\
&\cdots\\
&\cdots\\
&\cdots\\
C_1\frac{dv_{C_1}^{(5)}}{dt} &= G\left(v_{C_2}^{(5)} - v_{C_1}^{(5)}\right) - f\left(v_{C_1}^{(5)}\right)\\
C_2\frac{dv_{C_2}^{(5)}}{dt} &= G\left(v_{C_1}^{(5)} - v_{C_2}^{(5)}\right) + i_L^{(5)} + K_5\left(v_{C_2}^{(1)} - v_{C_2}^{(5)}\right)\\
L\frac{di_L^{(5)}}{dt} &= -v_{C_2}^{(5)}
\end{aligned}
$$

Each Chua's circuit [9], [10] contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor, and a single nonlinear resistor N_R, namely, Chua's diode [10] with a three-segment linear characteristic defined by

$$f(v_R) = m_0 v_R + \frac{1}{2}(m_1 - m_0)[|v_R + B_p| - |v_R - B_p|] \quad (2)$$

where the slopes in the inner and outer regions are m_0 and m_1, respectively, and $\pm B_p$ denotes the breakpoints. Each Chua's circuit is coupled to the next one in such a way that the difference between the signals $v_{C2}^{(i-1)}$ and $v_{C2}^{(i)}$

$$d(t) = K\left(v_{C_2}^{(i-1)} - v_{C_2}^{(i)}\right) \quad (3)$$

is introduced into each ith circuit as a negative feedback current. $K > 0$ is the stiffness of the perturbation, which we consider as a control parameter. In our experiments we took $C_1 = 10$ nF, $B_p = 1$ V, $C_2 = 99.34$ nF, $m_1 = -0.76$ mS, $m_0 = -0.41$ mS, $L = 18.46$ mH, $R = 1.64$ KΩ, i.e., we assume that each Chua's circuit operates on the chaotic double-scroll Chua's attractor [10], [11].

System (1) is a 15-dimensional dynamical system and its behavior is characterized by 15 Lyapunov exponents. Due to our assumption that each Chua's circuit operates on the chaotic double-scroll Chua's attractor, system (1) can have from one to five positive Lyapunov exponents depending on the value of coupling stiffness K_i. For experimental observation of hyperchaotic attractors we exploit some results from chaos synchronization theory [12]–[18]. When both Chua's circuits are operating in a chaotic regime, it is possible to achieve synchronization [8], [18] using the above coupling. It was shown by de Sousa *et al.* [14] that the boundary of the possible synchronization (according to the definition by Pecora and Carroll [12]) and nonsynchronization is strictly connected to the transition from chaotic to hyperchaotic behavior. This result can be generalized

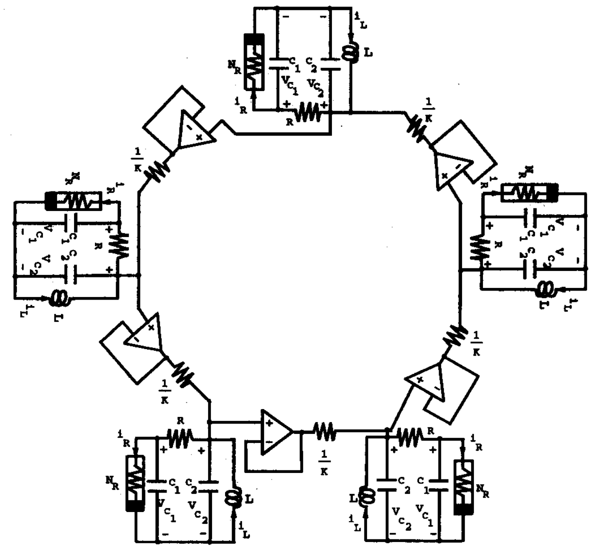


Fig. 1. Five identical coupled Chua's circuits forming a ring.

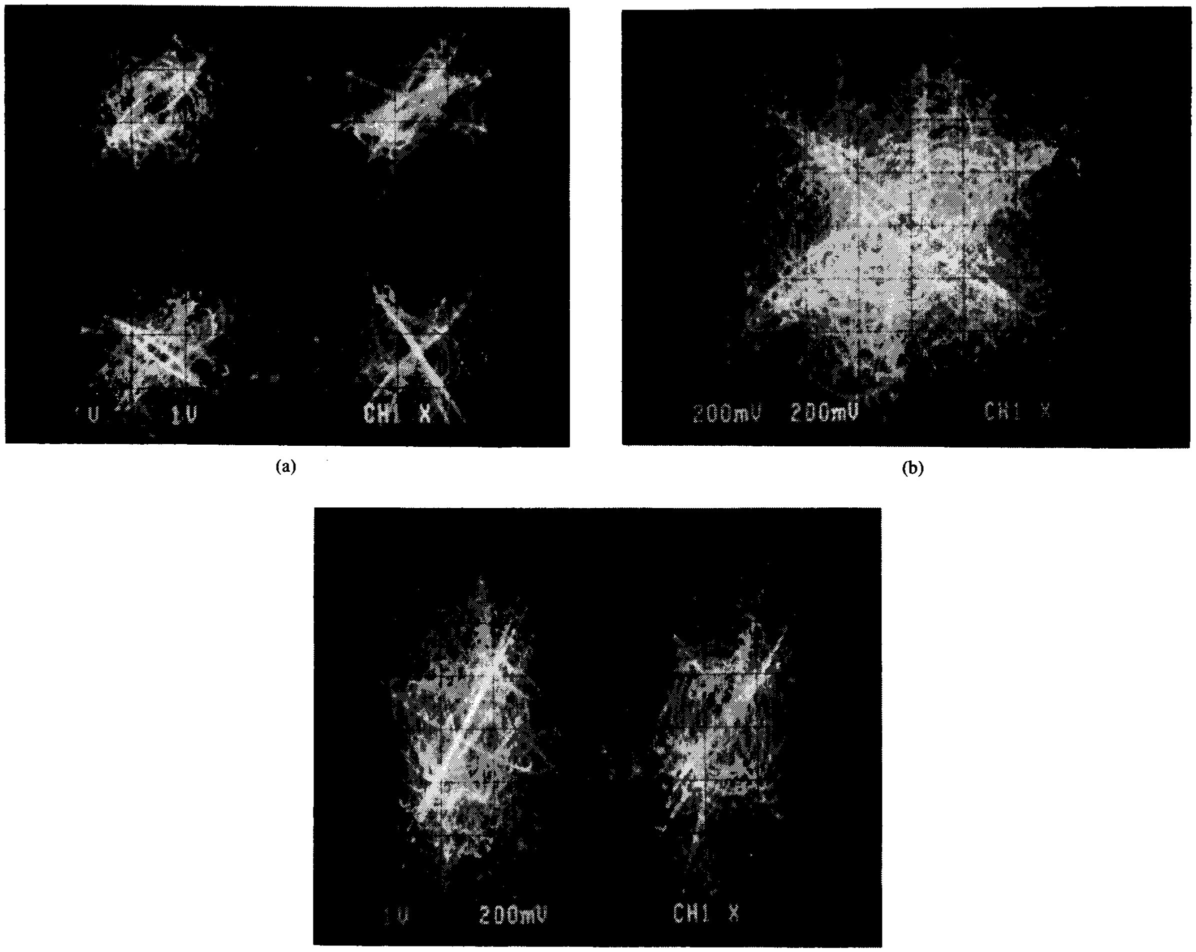
Manuscript received March 11, 1994. This paper was recommended by Associate Editor Hsiao-Dong Chiang.

T. Kapitaniak is with the Division of Control and Dynamics, Technical University of Lodz, Stefanowskiego 1/15, 90-924 Lodz, Poland.

L. O. Chua and G.-Q. Zhong are with the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720 USA.

IEEE Log Number 9402838.

1057-7122/94$04.00



(a) (b) (c)

Fig. 2. Experimental two-dimensional projections of hyperchaotic attractors: $K_{1-5} = 0.01$; (a) $v_{C1}^{(1)}$ versus $v_{C1}^{(3)}$, Horizontal axis is $v_{C1}^{(1)}$, 1 V/div; Vertical axis is $v_{C1}^{(3)}$, 1 V/div, (b) $v_{C2}^{(1)}$, versus $v_{C2}^{(3)}$, Horizontal axis is $v_{C2}^{(1)}$, 200 mV/div; Vertical axis is $v_{C2}^{(3)}$, 200 mV/div, (c) $v_{C1}^{(1)}$ versus $v_{C2}^{(3)}$, Horizontal axis is $v_{C1}^{(1)}$, 1 V/div; Vertical axis is $v_{C2}^{(3)}$, 200 mV/div.

to the following conjecture: If in a $3N$-dimensional chain of Chua's circuits (1) no two circuits synchronize with each other, then system (1) has a hyperchaotic attractor with N positive Lyapunov exponents. This property suggests that we observe the system behavior of $v_{C1}^{(j)}$ versus $v_{C1}^{(i)}$, where $i \not\simeq j, i, j = 1, 2, \ldots, 5$, plots. If two of the Chua's circuits i and j synchronize with each other, the plot $v_{C1}^{(i)}$ versus $v_{C1}^{(j)}$ will be a straight line. On the other hand, if all such plots exhibit complicated structures, we have a hyperchaotic attractor with five positive Lyapunov exponents.

In Fig. 2(a)–(c) we have shown some two-dimensional projections of the attractor of system (1) for the case where no two Chua's circuits synchronize with each other. Our experimental results are in good agreement with the numerical simulations of (1), as can be seen in Fig. 3 (a)–(c) where we have presented the simulated results corresponding to Fig. 2(a)–(c) plots obtained using the software INSITE [19]. Calculation of Lyapunov exponents showed that the attractors of Figs. 2 and 3 are characterized by five positive Lyapunov exponents: $\lambda_1 = 0.44$, $\lambda_2 = 0.43$, $\lambda_3 = 0.43$, $\lambda_4 = 0.41$, $\lambda_5 = 0.41$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = -0.1$, $\lambda_{10} = -0.1$, $\lambda_{11} = -3.60$, $\lambda_{12} = -3.69$, $\lambda_{13} = -3.73$, $\lambda_{14} = -3.79$, $\lambda_{15} = -3.88$. Hyperchaotic attractors with five positive, three zero and seven negative Lyapunov exponents are robust in system (1) and we can observe them for $K_i < 0.016$.

Hyperchaotic attractors can also be observed in the case of an open chain of unidirectionally coupled Chua's circuits (i.e., $K_5 = 0$ in (1)), as shown in Fig. 4. In this case, we also observed hyperchaotic attractors with five positive Lyapunov exponents for $K_{1-4} < 0.21$. For example, for $K_{1-4} = 0.01$ we have an attractor with the following Lyapunov exponents: $\lambda_1 = 0.43$, $\lambda_2 = 0.42$, $\lambda_3 = 0.41$, $\lambda_4 = 0.41$, $\lambda_5 = 0.40$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = 0$, $\lambda_{10} = 0$, $\lambda_{11} = -3.80$, $\lambda_{12} = -3.82$, $\lambda_{13} = -3.82$, $\lambda_{14} = -3.83$, $\lambda_{15} = -3.84$.

The difference in the numbers of zero and negative Lyapunov exponents in the spectrum of hyperchaotic attractors associated with the open and closed chains, respectively, of Chua's circuits suggests different topological structures of these attractors. Unfortunately, due to the high-dimensionality of our system we are unable to observe these differences on two-dimensional projections of the attractor: in both cases these projections are similar and indistinguishable. As an example compare Fig. 5 with Fig. 2(a).

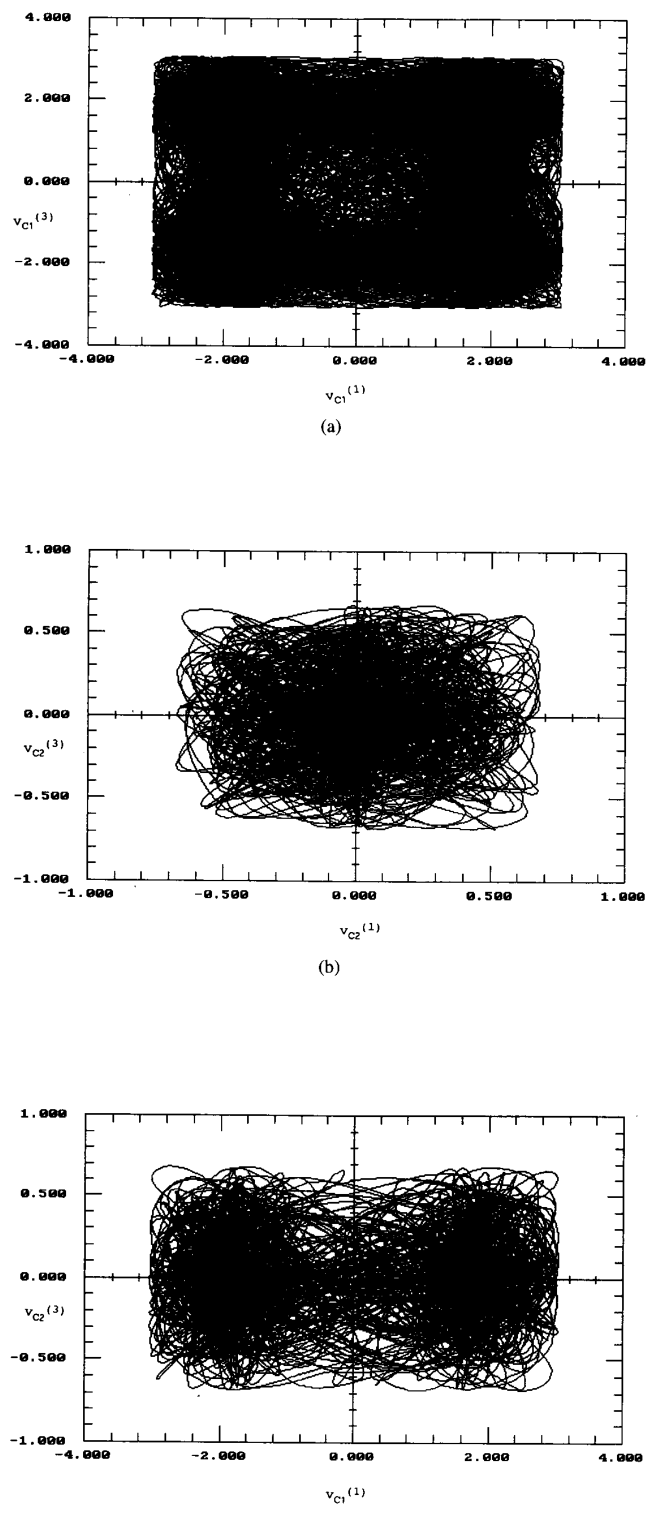


Fig. 3. Numerical two-dimensional projections of hyperchaotic attractors: $K_{1-5} = 0.01$; (a) $v_{C1}^{(1)}$ versus $v_{C1}^{(3)}$, (b) $v_{C2}^{(1)}$ versus $v_{C2}^{(3)}$, (c) $v_{C1}^{(1)}$ versus $v_{C2}^{(3)}$.

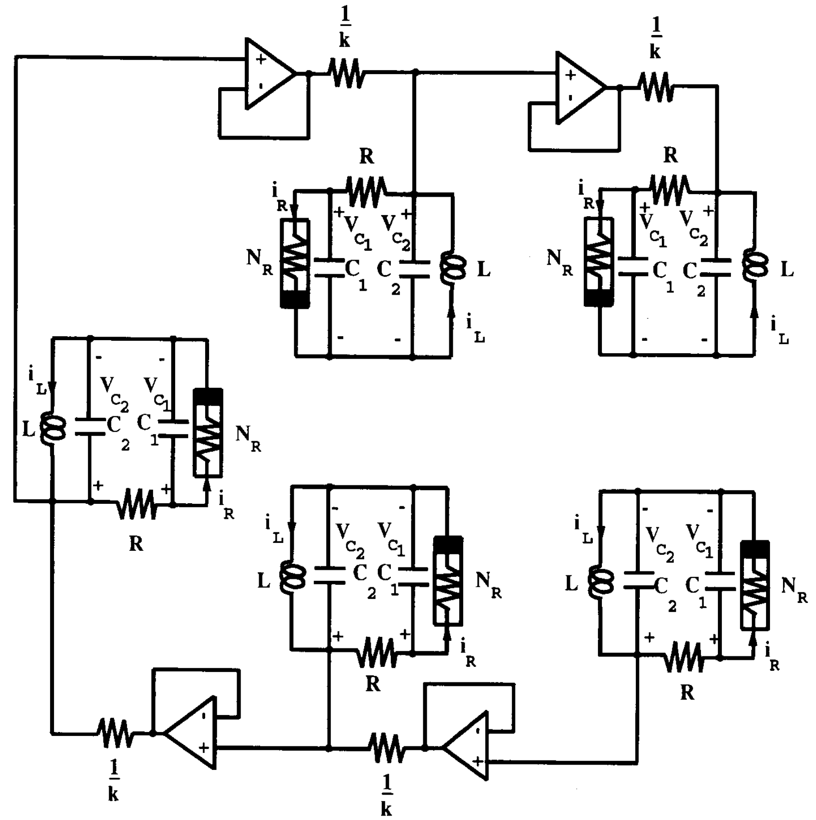


Fig. 4. Five identical unidirectionally coupled Chua's circuits.

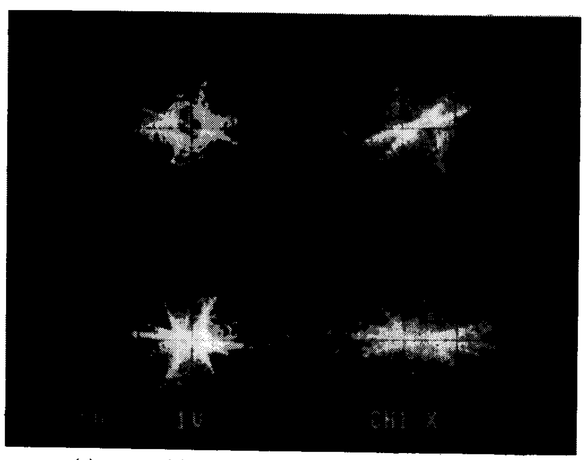


Fig. 5. $v_{C1}^{(1)}$ versus $v_{C1}^{(3)}$ plot of unidirectionally coupled Chua's circuits: $K_{1-4} = 0.01$, $K_5 = 0$, Horizontal axis is $v_{C1}^{(1)}$, 1 V/div; Vertical axis is $v_{C1}^{(3)}$, 1 V/div.

III. Summary

We have demonstrated experimentally that both open and closed chains of coupled Chua's circuits can exhibit hyperchaotic behaviors. In both cases hyperchaotic attractors are robust, but the Lyapunov exponents spectrum suggests different topological structures.

References

[1] P. Cvitanovic, Ed., *Universality of Chaos*, A. Hilger: Bristol, 1984.
[2] B.-L. Hao, Ed., *Chaos*. Singapore: World Scientific, 1986.
[3] T. Kapitaniak, Ed., *Chaotic Oscillators*. Singapore: World Scientific, 1992.
[4] O. E. Rossler, "An equation for hyperchaos," *Phys. Lett.*, vol. 71A, pp. 155–157, 1979.
[5] O. E. Rossler, "The chaotic hierarchy," *Z. Naturforsch.*, vol. 38a, pp. 788–801, 1983.
[6] T. Kapitaniak and W.-H. Steeb, "Transition to hyperchaos in coupled generalized van der Pol's equations," *Phys. Lett.*, vol. 152A, pp. 33–36, 1991.
[7] T. Kapitaniak, "Transition to hyperchaos in chaotically forced coupled systems," *Phys. Rev.*, vol. 47E, pp. 2975–2978, 1993.
[8] T. Kapitaniak and L. O. Chua, "Hyperchaotic attractors of unidirectionally-coupled Chua's circuits," *Int. J. Bifurcation and Chaos*, in press.
[9] L. O. Chua, M. Komuro, and T. Matsumoto, "The double scroll family parts I and II," *IEEE Trans. Circuits and Syst.*, vol. 33, no. 11, pp. 1073–1118, 1986.
[10] L. O. Chua, "Global unfolding of Chua's circuit," *IEICE Trans. Fundamentals*, vol. E76-A, pp. 704–734, 1993.
[11] M. P. Kennedy, "Robust op amp implementation of Chua's circuit," *Frequenz*, vol. 46, pp. 66–80, 1992.
[12] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, pp. 821–825, 1990.
[13] L. M. Pecora and T. L. Carroll, "Synchronizing chaotic circuits," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 453–458, 1991.
[14] M. de Sousa Vieira, A. J. Lichtenberg, and M. A. Lieberman, "Self-synchronization of many coupled oscillators," *Phys. Rev.*, vol. A46, pp. 7359–7362, 1992.
[15] V. S. Afraimovich, N. N. Verichev, and M. I. Rabinovich, "Stochastic

synchronization of oscillations in dissipative systems," *Radiophysics and Quantum Electron.*, vol. 29, pp. 795–803, 1986.
[16] V. S. Anishchenko, T. E. Vadivasova, D. E. Postnov, and M. A. Safonova, "Forced and mutual synchronization of chaos," *Radioeng. & Electr.*, vol. 36, pp. 338–351, 1991.
[17] T. Endo and L. O. Chua, "Synchronizing chaos from electronic phase-locked loops," *Int. J. Bifurcation and Chaos*, vol. 1, pp. 701–710, 1991.
[18] L. Kocarev, A. Shang, and L. O. Chua, "Transitions in dynamical regimes by driving: A uniform method of control and synchronization of chaos," *Int. J. Bifurcation and Chaos*, vol. 3, pp. 479–483, 1993.
[19] T. S. Parker and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems.* New York: Springer, 1989.
[20] T. Buzug, J. von Stamm, and G. Pfister, "Characterization of period doubling scenarios in Taylor-Couette flow," *Phys. Rev.* vol. E47, pp. 1054–1065, 1993.
[21] R. Stoop, J. Peinke, J. Parisi, B. Rohricht, and R. P. Huebener, "Ap-Ge semiconductor experiment showing chaos and hyperchaos," *Physica*, vol. D35, pp. 425–435, 1989.