Journal of Sound and Vibration (1994) **178**(2), 272–275

EXPERIMENTAL OBSERVATION OF INTERMITTENT CHAOS IN A MECHANICAL SYSTEM WITH IMPACTS

B. BŁAZEJCZYK, T. KAPITANIAK AND J. WOJEWODA

Division of Control and Dynamics, Technical University of Łódz, 90-924 Łódz, Poland

AND

R. BARRON

Division of Dynamics and Control, University of Strathclyde, Glasgow G1, Scotland

(*Received* 2 *December* 1993, *and in final form* 12 *April* 1994)

Intermittency is a type of chaotic behaviour commonly observed in deterministic systems [1–9]. It is characterized by long periods of regular motion interrupted by short chaotic bursts. When a burst starts at the end of a laminar phase this denotes an instability of the periodic motion due to the fact that the modulus of at least one Floquet multiplier is larger than one. As is well known [10], it may occur in three different ways: a real Floquet multiplier crosses the unit circle at $(+1)$ or at (-1) or two complex conjugate multipliers cross it simultaneously. In their pioneering work [1], Pomeau and Manneville associated to each of these typical crossings one type of intermittency. Type I is characterized by crossing at $(+1)$, type II by complex crossing and type III by crossing at (-1). If ω^* is a value of a control parameter ω for which the fully developed chaotic motion is observed and L is an average length of laminar interval then we have the scaling law $L \propto (\omega - \omega^*)^{-\gamma}$, where $\gamma = 1/2$ for type I and type III intermittency and $\gamma = 1$ for type II intermittency [10].

Intermittent chaos has been experimentally observed mainly in hydrodynamic systems [6, 7] and chemical systems [8, 9]. Up to now there have been no reports of experimental intermittent chaos in a purely solid mechanical system.

In this letter we consider a mechanical system as shown in Figure 1. Mass m can move on a bar C in direction x between two rigid stops A and B. After each collision with stops A or B the sign of $x(t)$ is changed. The whole system is connected with a dynamical shaker and forced by a sinusoidal force $F \cos \omega t$, where F and ω are respectively the amplitude and frequency of the forcing. During our experiments we measured the displacement of the mass m, $x(t)$, using a 12-bit data acquisition system in a microcomputer. Simultaneously, we registered the forcing frequency ω. The time series $x(t)$ allowed us to calculate a number of impacts, N in each period of excitation $T = 2\pi/\omega$. It should be noted here that the number of impacts per excitation period factor has been successfully used in a description of non-linear behaviour of systems with impacts in numerical analysis [11, 12]. Typical time series $N(nT)$, where $n = 1, 2, \ldots$, are shown in Figure 2. Complete description of the dynamics of the systems described will be given elsewhere. Here we only present the possibility of experimental intermittent chaos in our system.

The analysis of a time series of Figure 2(a) shows that the behaviour of the system for $\omega = 11$ Hz is characterized by long intervals with regular behaviour (two impacts per period T). This behaviour is occasionally interrupted with T-periods with zero, one or three impacts. With the increase of ω to 12 Hz (see Figure 2(b)) the length of regular two-impact intervals decreases and periods with zero, one or three impacts become more frequent. With further increase of excitation frequency ω to 14 Hz (see Figure 2(c)) the

0022–460X/94/470272 + 04 $08.00/0

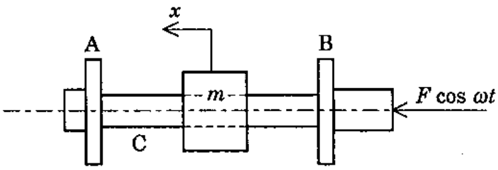

Figure 1. A model of a mechanical system with impacts.

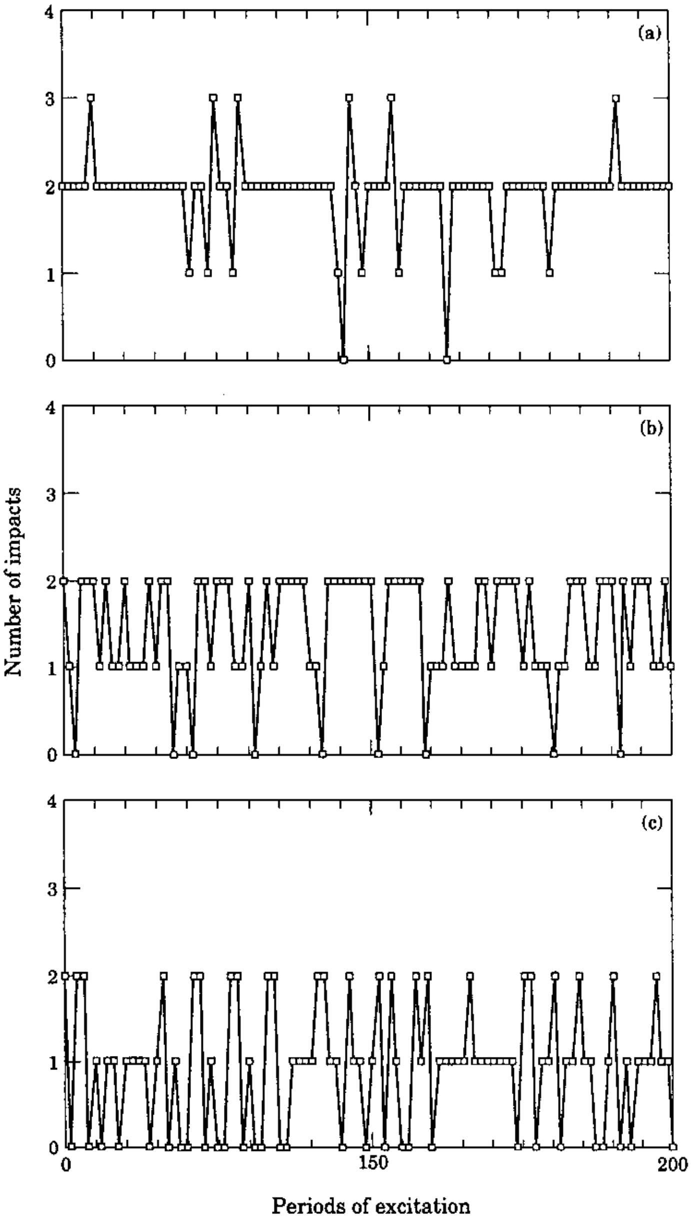

Figure 2. $N(nT)$ time series: (a) ω = 11 Hz; (b) ω = 12 Hz; (c) ω = 14 Hz.

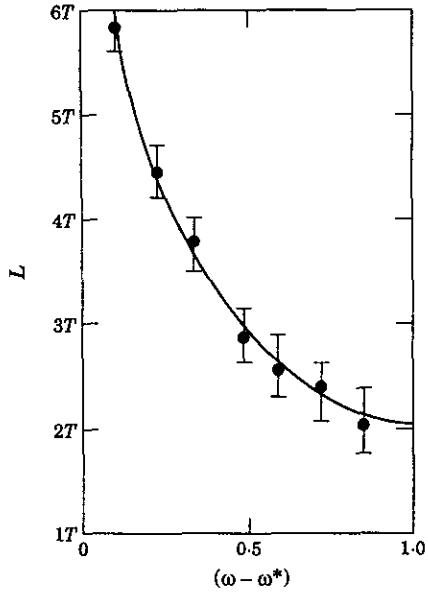

Figure 3. A plot of L *vs.* $(\omega - \omega^*)$: ⧳, experimental results; ——, mean square approximation.

regular two-impact periods are not visible at all and we have fully developed chaotic behaviour.

If we consider an average length of laminar interval versus forcing frequency ω, then the mean square approximation shown in Figure 3 gives us $L \propto (\omega - \omega^*)^{-1\cdot 03 + 0\cdot 02}$, where $\omega^* = 10\cdot 25$ Hz was taken as a threshold for chaos.

The scaling factor γ is very close to 1, which is a characteristic value for type II intermittency. As our experimental system was not free of external noise, it seems that the chaotic motion that we observed is a good candidate for type II intermittency.

To summarize, in this letter we have presented an example of experimental intermittent chaos which, to our knowledge, is the first example of such behaviour in a purely solid mechanical system.

REFERENCES

1. Y. Pomeau and P. Manneville 1980 *Communications in Mathematical Physics* **74**, 189–197. Intermittent transition to turbulence in dissipative dynamical systems.
2. T. Kohyama and Y. Aizawa 1984 *Progress in Theoretical Physics* **71**, 917–929. Theory of intermittency chaos.
3. R. Benzi, G. Paladin, G. Parisi and A. Vulpiani 1984 *Journal of Physics A* **17**, 3521–3525. Characterization of intermittency chaos.
4. B. Hu and J. Rudnick 1982 *Physical Review A* **28**, 1645–1649. Exact solution of the Feigenbaum renormalization group equations for intermittency.
5. J. E. Hirsch, M. Nauenberg and D. J. Scalapino 1982 *Physics Letters* **87A**, 391–398. Intermittency in the presence of noise: a renormalization group formulation.
6. P. Berge, M. Dubois, M. Manneville and Y. Pomeau, 1980 *Journal of Physics Letters* (*Paris*) **41**, L341–L346. Intermittency in Rayleigh–Bernard convection.
7. E. Ringuet, C. Roze and G. Gouesbet 1993 *Physical Review* **47E**, 1405–1407. Experimental observation of type-II intermittency in a hydrodynamic system.
8. Y. Pomeau, J. C. Roux, A. Rossi, S. Bachelart and C. Vidal 1981 *Journal of Physics Letters* (*Paris*) **42**, L271–L274.

9. H. HAUCKE, R. E. ECKE, Y. MAENS and J. C. WHEATLEY 1984 *Physical Review Letters* **53**, 2090–2094. Noise induced intermittency in a convecting dilute solution of ^{3}He in superfluid ^{4}He.
10. E. OTT 1993 *Chaos in Dynamical Systems*. Cambridge: Cambridge University Press.
11. H. ISOMAKI 1985 *Physics Letters*, **107A**, 343–347. Devil's attractor and chaos in a driven impact ocillator.
12. J. M. T. THOMPSON and R. GHAFFARI 1982 *Physics Letters* **91A**, 5–8. Chaos after period-doubling bifurcation in the resonance of an impact oscillator.