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INSTABILITIES OF THE VISCO-ELASTIC ROTOR SUBJECTED TO A FOLLOWER FORCE

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Rotors subjected to follower forces can be found in many industrial machines, such as compressors, pumps and turbines. The follower force has a non-conservative component perpendicular to the axis of the bearings, which changes over time. Such a load of the rotor may change the dynamic characteristic of the rotor motion to a considerable extent. Vibration and stability problems of elastic systems subjected to non-conservative forces have received considerable attention in modern rotor design.

A physical model of the rotor subjected to the follower force is shown in Figure 1. The shaft of the rotor of flexural rigidity EJ and constant cross-section is supported at its ends by two non-linear isotropic bearings. The non-linear character of the support has been assumed to be represented by the well-known Van der Pol and Duffing terms. A compressive follower force P of constant magnitude acts on the disk of the rotor. At the left end there is a thrust bearing subjected to the axial component of the follower force. The force P compresses only the part of the rotor between the disk and the thrust bearing. The angular velocity of the rotor, ω, is constant. The x-axis determines the static equilibrium position of the shaft.

To find the mathematical model of the rotor, the finite element method has been used as the modeling procedure. The reactions of the rotor support have been taken into consideration in the form

$$F_s = -ky_s - k_1 y_s^3 - c\dot{y}_s + c_1 y_s^2 \dot{y}_s, \tag{1}$$

where y_s is the lateral displacement of the shaft in the support position, k and c are the coefficients of the linear stiffness and damping respectively, k_1 and c_1 are the coefficients of the non-linear stiffness and damping respectively.

Recently, the stability analysis of such a system without damping has been presented [1]. In this letter we would like to direct attention to some instabilities which can occur when considering damping in the rotor system.

In the visco-elastic linearized system subjected to the follower force, two types of instability, divergence and flutter, may occur. The divergence instability occurs when

$$\alpha_r + \mathrm{j}\alpha_i = 0 + \mathrm{j}0, \tag{2}$$

where $\alpha = \alpha_r + \mathrm{j}\alpha_i$ is one of the complex eigenfrequencies. The flutter instability occurs when

$$\alpha_r + \mathrm{j}\alpha_i = 0 + \mathrm{j}\alpha_f, \qquad \alpha_f \neq 0. \tag{3}$$

The lowest positive value of the divergence or flutter load is the critical load of the linearized rotor system.

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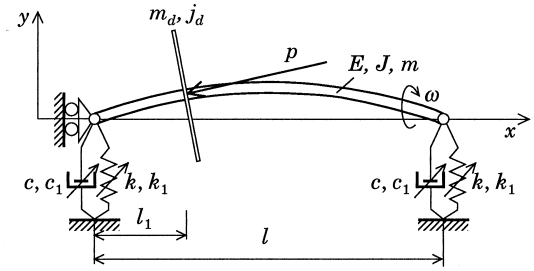

Figure 1. A model of the rotor.

To complete earlier investigations some examples of results of numerical calculations of the critical rotor's load are presented for the visco-elastic linearized system. For the disk positions $l_1/l = 0{\cdot}1$ (Figure 2) and $l_1/l = 0{\cdot}2$ (Figure 3), in the range of dimensionless stiffness and damping considered, only flutter instability of the rotor motion can be observed. When the linear stiffness of the bearing increases, the critical force depends less and less on the value of the support damping.

The change of the type of instability can be observed for the disk position $l_1/l = 0{\cdot}5$ (Figure 4), when the disk is located in the middle of the shaft. The divergence instability occurs for the dimensionless bearing stiffness $k > 200$. In this case the critical force is practically independent on the support stiffness and damping.

The investigation method of the non-linear rotor system has been derived from the Hopf bifurcation theory due to Iooss and Joseph [2]. The method consists in seeking the approximate periodic solutions of the non-linear equation of motion in a parametric form by using the Fredholm alternative. The investigations were concentrated on the behavior of free vibrations in the neighborhood of some points on the stability boundary in the flutter instability region of the linearized system. The character of the Hopf bifurcation (sub- or supercritical) and the limit cycle of the shaft have been investigated.

Some example of results of the investigations are presented for the fixed position of the disk ($l_1/l = 0{\cdot}2$) and for different values of support parameters. The plot of the estimated radius of the bifurcation solution in the neighborhood of the stability threshold versus the critical force is shown in Figure 5. For $k < 125$ the subcritical Hopf bifurcation with an unstable limit cycle can be observed. Above this stiffness the region of supercritical Hopf bifurcation with a stable limit cycle occurs. Results of investigations show that the type of bifurcation depends on the linear parameters of the support. However, both linear and non-linear terms of the support stiffness influence the radius of the limit cycle. Additionally, a change of the Duffing term sign changes the type of bifurcation: from subcritical to supercritical and *vice versa*.

The next example shows the influence of the Van der Pol term c_1. In Figure 6 is shown the position of the boundary between subcritical and supercritical Hopf bifurcation for two values of non-linear damping. Dotted lines in Figure 6 show the stability threshold of the linearized system for different values of linear parameters of the support. The change of the sign of the non-linear damping considerably changes the boundary position between the subcritical and supercritical bifurcation. For instance, for $c = 30$ (the same value as in Figure 5) and for $c_1 = -10^4$, in the range of support parameters considered, only the subcritical Hopf bifurcation (Figure 7) has been found.

These examples show a high sensitivity of the behavior of free vibrations in the flutter instability region to changes of the support's and disk's parameters. In a more developed and more complicated model of the rotor system (anisotropic support, non-uniform mass of the shaft, many degrees of freedom, etc), this sensitivity can be observed to a much

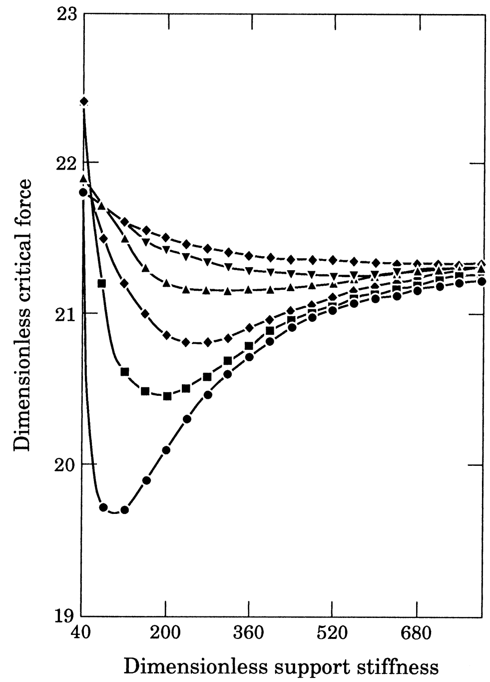

Figure 2. Stability thresholds, $l_1/l = 0{\cdot}1$. c values: —●—, 30; —■—, 150; —◆—, 270; —▲—390; —▼—, 510; —◆—, 630.

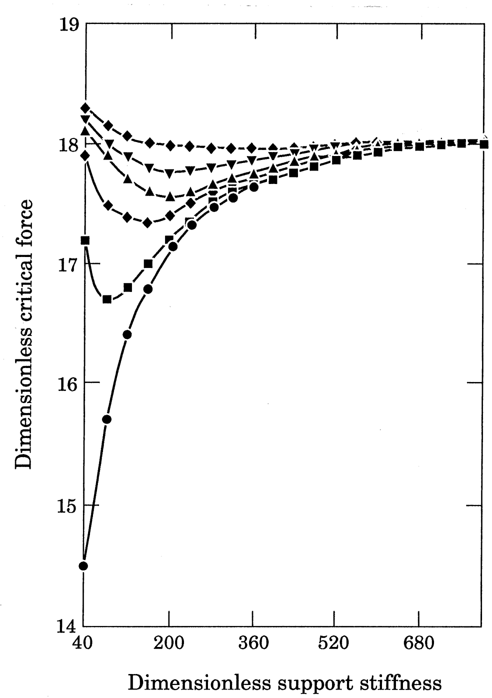

Figure 3. Stability thresholds, $l_1/l = 0{\cdot}2$. Key as Figure 2.

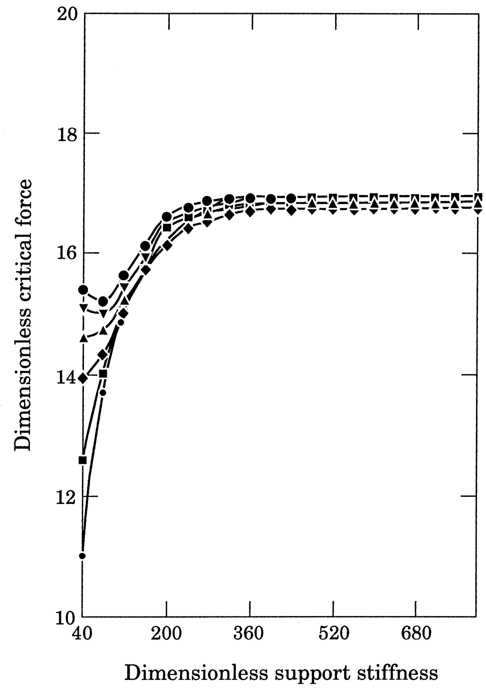

Figure 4. Stability thresholds, $l_1/l = 0{\cdot}5$. Key as Figure 2 except —●—, $c = 630$.

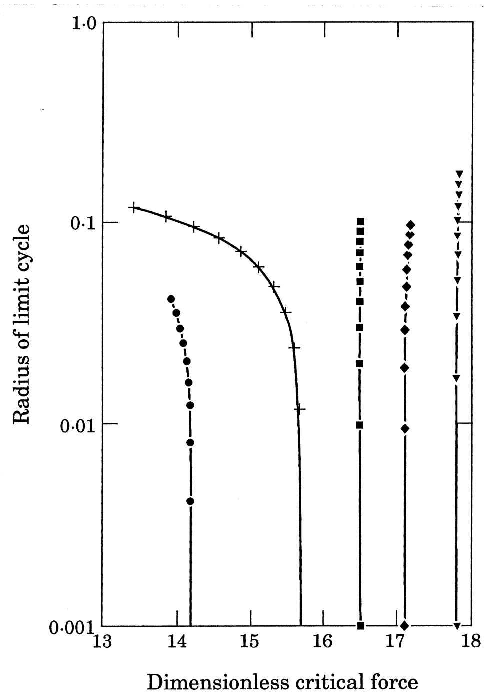

Figure 5. Bifurcation diagram of nonlinear rotor; $l_1/l = 0{\cdot}2$, $c = 30$, $k_1 = 10^4$, $c_1 = 0$. k values: —●—, 20; +, 80; —■—, 125; —◆—, 200; —▼—, 500.

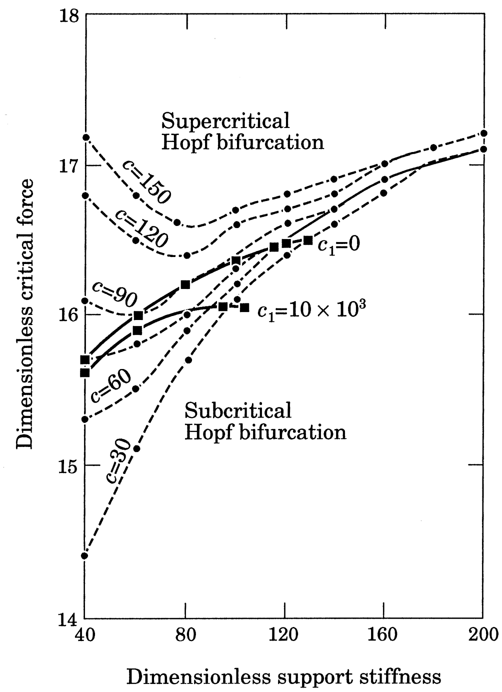

Figure 6. Boundaries of sub- and supercritical Hopf bifurcation; $l_1/l = 0{\cdot}2$, $k_1 = 10^4$; ---, stability thresholds of linearized system.

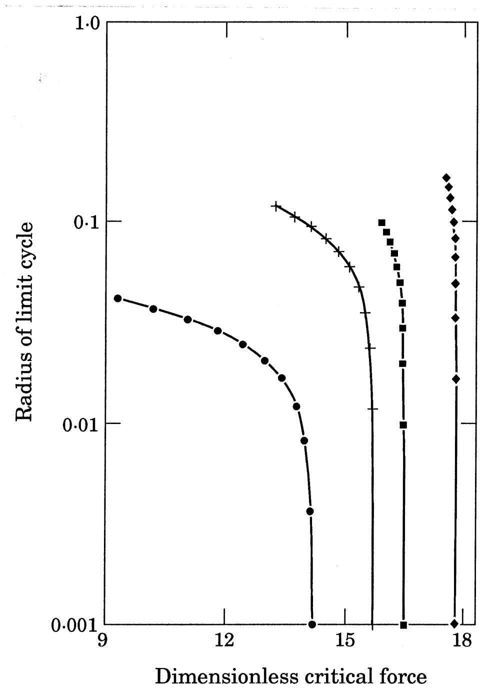

Figure 7. Bifurcation diagram of non-linear rotor; $l_1/l = 0{\cdot}2$, $c = 30$, $k_1 = 10^4$, $c_1 = -10^4$. Key as Figure 5, except —◆—, $k = 500$.

greater extent. Under these circumstances it is possible that two bifurcations may occur simultaneously. This codimension two bifurcation creates a quasi-periodic response of the rotor, and under some perturbations can lead to chaotic motion. More details of this problem will be given elsewhere [3].

From the practical point of view, the results of the numerical simulations indicate that the bifurcation analysis of free vibrations should be taken into account in all new designs of machine elements which are subjected to non-conservative forces. Otherwise, besides other bifurcations, a very dangerous subcritical Hopf bifurcation with an unstable limit cycle may appear when the new machine is operating. The existence of this unstable limit cycle can lead to transient vibrations with large amplitude which can damage the machine. It should be noted that the existence of a subcritical Hopf bifurcation cannot be determined by classical stability analysis.

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