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Steady state locking in coupled chaotic systems

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Abstract

Two Lorenz systems working in different chaotic ranges can be stabilized simultaneously in the same steady state by coupling them through the negative feedback mechanism. This kind of locking is robust as it can be realized for a wide range of parameters.

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Recently it has been demonstrated that two identical chaotic systems $\dot{x}=f(x)$ and $\dot{y}=f(y)$ $(x,\ y\in\mathbb{R}^n,\ n\geqslant 3)$ coupled with each other can be synchronized [1–6]. Coupling of homochaotic systems (i.e. systems given by the same set of ODEs but with different values of the system parameters) can lead to the practical synchronization (i.e. $x\neq y$, but $\|x-y\|\leqslant\epsilon$ where ϵ is a vector of small parameters) [7–9]. In such coupled systems we can also observe a significant change of the chaotic behavior of one or both systems [11,12]. This so-called "controlling chaos by chaos" procedure has some potential importance in a range of contexts varying from mechanical and electrical systems [7], where control of the system is the objective, to geophysical systems, like the atmosphere or oceans, where improvement in basic understanding and prediction is the main motivation [11,12]. Other phenomena are possible as the coupled systems become a new augmented system, which has its own dynamics. In Ref. [10] it was shown that two Lorenz systems working in different chaotic ranges can be stabilized simultaneously in different periodic orbits by coupling them through certain system parameters.

In what follows we introduce another phenomenon characteristic for such systems. We consider two monochaotic systems

$$\dot{x}=f(a_1,\ x) \tag{1}$$

and

$$\dot{y}=f(a_2,\ y) \tag{2}$$

$(x,\ y\in\mathbb{R}^n,\ a_{1,2}\in\mathbb{R},\ n\geqslant 3)$ coupled with each other by negative feedback. The augmented system is as follows,

$$\dot{x}=f(a_1,\ x)+k_1(y-x),\qquad \dot{y}=f(a_2,\ y)+k_2(x-y), \tag{3}$$

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where $k_{1,2} = [k_{1,2}, k_{1,2}, \dots, k_{1,2}]^{\mathrm{T}} \in \mathbb{R}^n$ is a coupling vector. We show that system (3) can be locked to the steady state given by the relations

$$f(a_1, x) + k_1(y - x) = 0, \qquad f(a_2, y) + k_2(x - y) = 0. \tag{4}$$

This kind of locking can be interesting with respect to suppressing chaos and may have some practical applications.

As an example we consider two Lorenz systems coupled by a one-to-one negative feedback mechanism,

$$\dot{X}_1 = -\sigma X_1 + \sigma Y_1 + k_1(X_2 - X_1), \tag{5a}$$

$$\dot{Y}_1 = -X_1 Z_1 + r_1 X_1 - Y_1 + k_1(Y_2 - Y_1), \tag{5b}$$

$$\dot{Z}_1 = X_1 Y_1 - bZ_1 + k_1(Z_2 - Z_1), \tag{5c}$$

$$\dot{X}_2 = -\sigma X_2 + \sigma Y_2 + k_2(X_1 - X_2), \tag{5d}$$

$$\dot{Y}_2 = -X_2 Z_2 + r_2 X_2 + k_2(Y_1 - Y_2), \tag{5e}$$

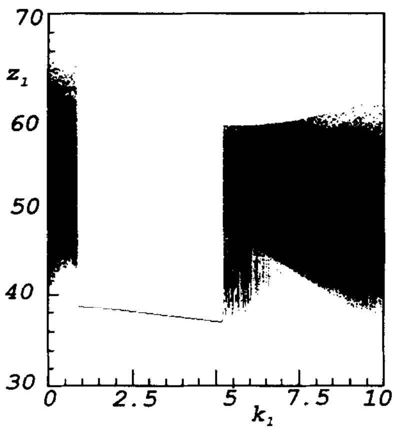
$$\dot{Z}_2 = X_2 Y_2 - bZ_2 + k_2(Z_1 - Z_2), \tag{5f}$$

where σ, b, $r_{1,2}$, k_1 and k_2 are constants. The Lorenz model [13] has been often proposed as a paradigm for chaos. Two coupled Lorenz models introduced in Eq. (5) can be considered as models of geophysical flows [12].

All our numerical computations have been carried out using the software INSITE [14]. We considered the following parameter values: $\sigma = 10.0$, $b = 8/3$, $r_1 = 40.0$ for the first system (Eqs. (5a)–(5c)) and $r_2 = 30.0$ for the second one (Eqs. (5d)–(5f)). In the case of $k_1 = k_2 = 0$ (two separate Lorez models) both systems have a chaotic attractor. In Fig. 1 we show the bifurcation diagram of the coupled system (5) for $k_2 = 2.0$, with k_1 as a control parameter. The analysis of this diagram shows that for small values of k_1 both systems are chaotic. With the increase of k_1 we can observe windows where both systems are locked to the steady state given by Eq. (4).

To analyse the stability of the steady state we performed a linear stability analysis. Eqs. (5) have been linearized in the form

$$\dot{X} = AX, \tag{6}$$



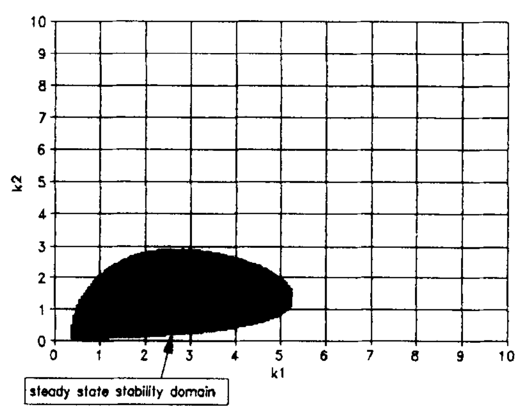
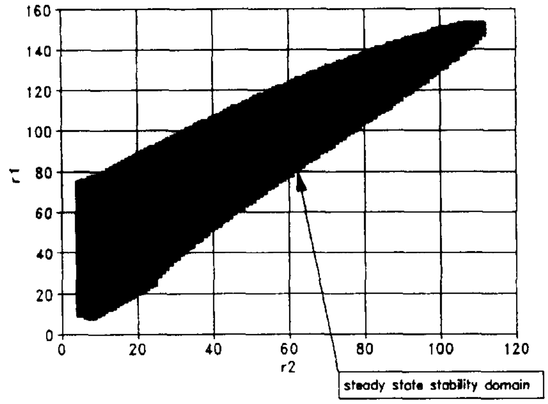


Fig. 1. Bifurcation diagram of Eqs. (5); $\sigma = 10.0$, $b = 8/3$, $r_1 = 40.0$, $r_2 = 30.0$, $k_2 = 2.0$.

Fig. 2. Stability domain of the steady state in the $k_1 - k_2$ plane; $\sigma = 10.0$, $b = 8/3$, $r_1 = 40.0$, $r_2 = 30.0$.



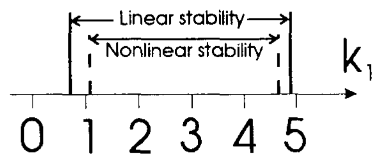


Fig. 3. Stability domain of the steady state in the $r_1 - r_2$ plane; $\sigma = 10.0$, $b = 8/3$, $k_1 = k_2 = 2.0$.

Fig. 4. Comparison of the linear and nonlinear steady state stability analyses; $\sigma = 10.0$, $b = 8/3$, $r_1 = 40.0$, $r_2 = 30.0$, $k_2 = 2.0$.

where $X = [X_1, Y_1, Z_1, X_2, Y_2, Z_2]^T$ and

$$A = \begin{bmatrix} -\sigma + k_1 & \sigma & 0 & k_1 & 0 & 0 \\ r_1 - Z_{1s} & -(1+k_1) & -X_{1s} & 0 & k_1 & 0 \\ Y_{1s} & X_{1s} & -(b+k_1) & 0 & 0 & k_1 \\ k_2 & 0 & 0 & -(b+k_2) & \sigma & 0 \\ 0 & k_2 & 0 & r_2 - Z_{2s} & -(1+k_2) & -X_{2s} \\ 0 & 0 & k_2 & Y_{2s} & X_{2s} & -(b+k_2) \end{bmatrix}, \tag{7}$$

where X_{1s}, Y_{1s}, Z_{1s}, X_{2s}, Y_{2s} and Z_{2s} are steady states given by Eq. (4).

Analysis of the eigenvalues of matrix A allows us to determine the stability domain of the steady state in the coupling parameter plane k_1–k_2 and the system parameter plane r_1–r_2, shown in Fig. 2, where it can be seen that steady state behavior is stable in a quite large range of coupling coefficients and in Fig. 3. In this case we fixed $k_1 = k_2 = 2.0$. Again we observe that the steady state behavior is stable for a large range of parameters r_1 and r_2.

The above linear analysis has been compared with a nonlinear analysis based on the estimation of Lyapunov exponents. The steady state is stable when all Lyapunov exponents of Eqs. (5) are negative. The results of this comparison are shown in Fig. 4 where it is seen that the linear analysis gives a good approximation of the stability domain.

In conclusion, we show steady state locking in two homochaotic systems. In two initially chaotic systems chaos is suppressed and they are locked to the same steady state. This locking is observed for a wide range of system parameters. Our results show that this kind of locking is robust in many chaotic systems.

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