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Dynamics of Impact Oscillator with Dry Friction

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Abstract—Our investigations of the dynamics of a simple mechanical system show that (i) the structure of the bifurcation diagram seems to be independent of the friction models, and (ii) small changes of experimentally estimated parameters like friction and restitution coefficients (within the error of estimation) can lead to significant qualitative changes in the system's dynamics. The last observation shows that for such systems it is very difficult (if not impossible) to build mathematical models which can qualitatively describe experimental results for all possible values of system parameters. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Impact oscillators (i.e. systems that have an oscillating object that impacts frequently with some other object or objects) occur in many technical situations. A classical example occurs in loose fitting joints which are designed in most mechanical devices to allow for thermal expansion. Practical importance and interesting dynamical features have caused the growing interest in such systems [1–17]. Initial studies [1–7] showed the rich bifurcation behaviour in simple mainly piece-wise linear oscillators excited sinusoidally. Later studies [8–15] have concentrated on the so-called grazing bifurcations, i.e. bifurcations observed at the bordering state between the impacting and non-impacting motion. In [16] we showed the evidence of the Π-type intermittency in an experimental impact oscillator.

In this paper, we consider the simple physical system shown in Fig. 1. The mass m_1 is connected to a vibrator giving sinusoidal force $F_0 \cos \omega t$ through the spring-damper system with stiffness coefficient k_1 and damping coefficient c_1. The second mass m_2 is placed on mass m_1 and its movement is limited by two borders A and B. The motion of mass m_2 on mass m_1 is influenced by friction force F_T. The purpose of this paper is to analyze the influence of (i) different models of friction force, and (ii) small changes of friction and restitution coefficients on the dynamics of considered system.

In [16] we presented a limited experimental study of the system of Fig. 1. The main result of [16] was the identification of the II-type intermittency for $\eta = 1.26$. One of the purposes of this paper is to compare previous experimental results [16] with computer model simulations.

2. MATHEMATICAL MODEL AND ITS PHYSICAL QUANTITIES

The model to be considered can be described by the following equations:

$$\begin{aligned}
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= \cos \eta\tau - b_1 y_2 - y_1 + f_T - b_1 \delta (y_2 - y_4), \\
\dot{y}_3 &= y_4, \\
\dot{y}_4 &= (F_T/\mu) + (\delta/\mu) b_1 (y_2 - y_4),
\end{aligned} \tag{1}$$

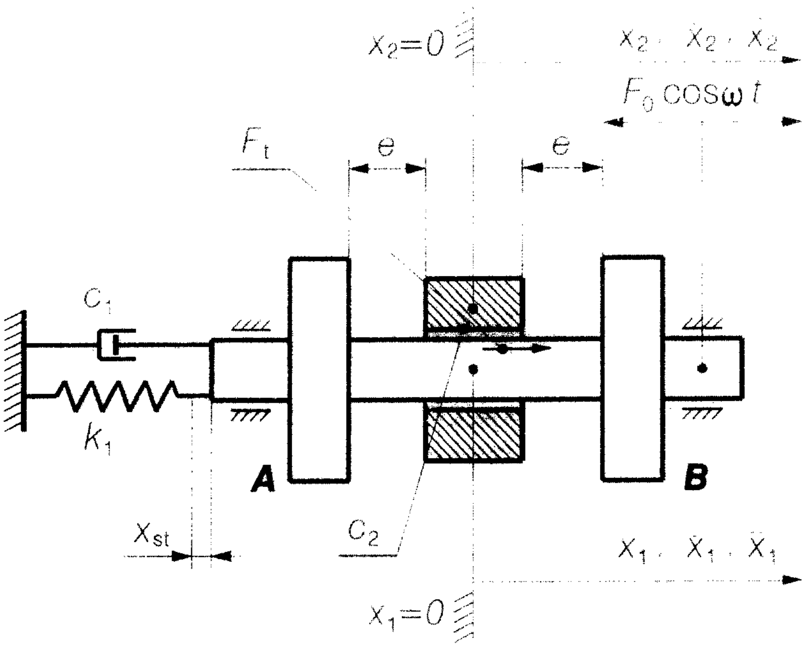

Fig. 1. Model of impact oscillator.

where $\tau = \Omega_1 t$, $\Omega_1 = \sqrt{k_1/m_1}$, $\eta = \omega/\Omega_1$, $x_{st} = m_2 g/k_1$, $b_1 = c_1/\Omega_1$, $b_2 = c_2/\Omega_1$, $y_1 = x_1/x_{st}$, $y_2 = \dot{x}_1$, $y_3 = x_2/x_{st}$, $y_4 = \dot{x}_2$, $\mu = m_2/m_1$, $\delta = c_2/c_1$, $f_T = F_T/m_2 g$ and g is gravitational acceleration. In most of our numerical investigations we took η as a control parameter and considered those values of η for which the motion of mass m_2 is characterized by impacts. Impactless motion and the motion in which the mass m_2 is not moving in relation to the mass m_1 is described in [17].

The impact conditions are given by the relation

$$|y_3 - y_1| \geq r, \tag{2}$$

where $r = e/x_{st}$ and

$$\begin{aligned} \dot{y}_{2(+)} &= \frac{m_1 y_{2(-)} + m_2 y_{4(-)}}{m_1 + m_2} - \frac{m_2}{m_1 + m_2} R(y_{2(-)} - y_{4(-)}), \\ \dot{y}_{4(+)} &= \frac{m_1 y_{2(-)} + m_2 y_{4(-)}}{m_1 + m_2} - \frac{m_2}{m_1 + m_2} R(y_{2(-)} - y_{4(-)}), \end{aligned} \tag{3}$$

where signs (+) and (−) indicate respectively velocities after and before impact, and R is the restitution coefficient.

To describe the dry friction force we considered two different models: a linear model

$$f_T = \lambda \,\mathrm{sgn}\,(y_2 - y_4), \tag{4}$$

where λ is a friction coefficient dependent on the surface in contact [18], and a nonlinear model

$$f_T = \left(\frac{\mu_0 - \mu_1}{1 + \lambda_1 |y_2 - y_4|} + \mu_0 + \lambda_2 (y_2 - y_4)^2\right) \mathrm{sgn}\,(y_2 - y_4), \tag{5}$$

where μ_0, μ_1, λ_1 and λ_2 are experimentally estimated constants characteristic for the surfaces in contact [19].

3. NUMERICAL RESULTS

The bifurcation diagram of system (1) with friction force (4) and typical system parameters $\lambda = 0.02$, $r = 0.8$, $\delta = 0.5$, $R = 0.6$, $b_1 = 0.1$ and $\mu = 0.693$ is shown in Fig. 2. For frequency $\eta \in [1.0, 1.25]$ we observe chaotic behaviour with typical windows. At $\eta = 1.26$ we have crisis in which chaos is replaced by period-four orbit. Although small chaotic intervals are present around $\eta = 1.62$, periodic behaviour is observed up to the value $\eta = 1.67$ at which we have Hopf bifurcation of the bifurcation diagram or secondary Hopf (Neimark) bifurcation of the system trajectory. As a result of this bifurcation, we observe quasi-periodic motion up to the value $\eta = 1.77$. For larger values of η small intervals of chaotic and periodic motions characterize the system's behaviour.

For the same parameters but considering nonlinear friction model (5) with $\mu_0 = 0.25$, $\mu_1 = 0.05$, $\lambda_1 = 1.42$ and $\lambda_2 = 0.005$ (these were chosen so that its mean square linearization gives linear model (4) with $\lambda = 0.02$), we obtained the bifurcation diagram shown in Fig. 3. The comparison of the two diagrams shows strong sensitivity of the qualitative behaviour on the friction model in some η-parameter regions. For example, for $\eta \in [1.2, 1.25]$ the linear model results indicate chaotic behaviour while the nonlinear models show periodic motion. In the two diagrams the sequence of main bifurcation is the same and seems to be independent of the friction model.

In experimental systems it is very difficult to accurately estimate the friction coefficients

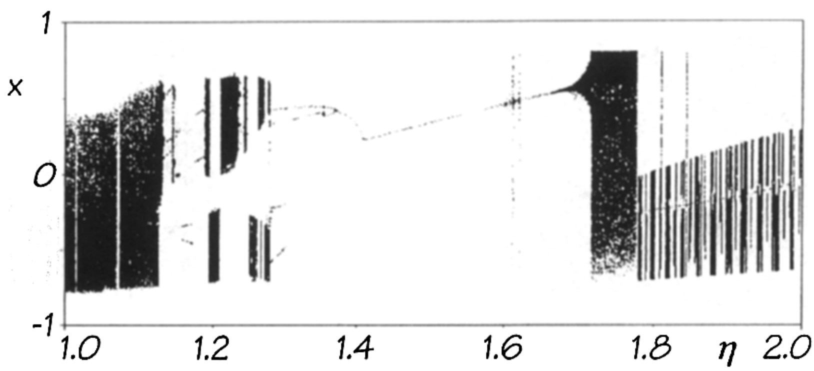

Fig. 2. Bifurcation diagram of system (1) with linear friction force (4): $\lambda = 0.02$, $r = 0.8$, $\delta = 0.5$, $R = 0.6$, $b_1 = 0.1$ and $\mu = 0.693$.

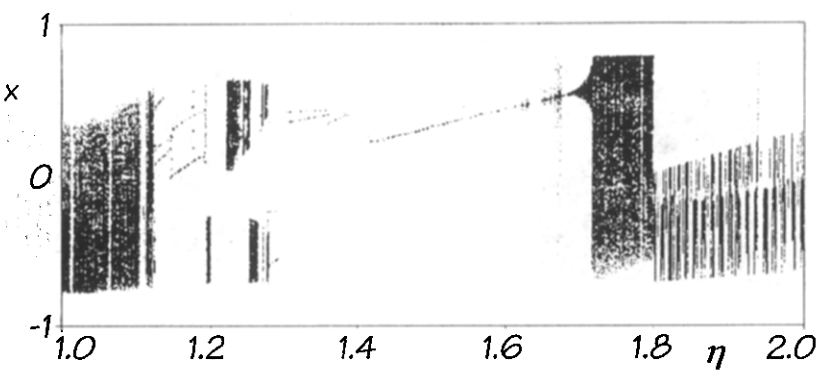

Fig. 3. Bifurcation diagram of system (1) with nonlinear friction force (5): $\mu_0 = 0.25$, $\mu_1 = 0.05$, $\lambda_1 = 1.42$, $\lambda_2 = 0.005$, $r = 0.8$, $\delta = 0.5$, $R = 0.6$, $b_1 = 0.1$ and $\mu = 0.693$.

λ, μ_0, μ_1, λ_1 and λ_2, and the restitution coefficient R. These parameters are usually given with an inaccuracy of a few per cent [20]. For the mechanical system working in or close to the chaotic regime these inaccuracies can qualitatively change the system's behaviour, as is shown in Fig. 4(a) and (b), where we repeated previous calculations with a slight change of restitution coefficient (now $R = 0.62$ is considered). The bifurcation diagram of Fig. 4(a) was obtained for linear friction force (4) and the diagram of Fig. 4(b) for a nonlinear model (5). The comparison of bifurcation diagrams shows that small changes of restitution coefficient (less than 4% in our case) have qualitative influence on the system's behaviour in some parameter regions, but the sequence of bifurcations is preserved again. Similar effects can be observed if we vary the friction parameters λ, μ_0, μ_1, λ_1 and λ_2 [17].

Unfortunately, the comparison of current numerical results with the experimental results shown in [16] does not give satisfactory agreement. For the frequency $\eta = 1.26$ for which in the experiment we observed the II-type intermittency, our numerical results indicate another type of chaotic or even periodic motion. As the considered system is very simple we can argue that the observed inaccuracies are caused by the inabilities of accurate estimations of the friction and restitution coefficients rather than by disadvantages in our mathematical model. This shows that it is impossible (at least, very difficult as it requires new more advanced friction and impact models) to have a mechanical system with chaotic motion for which there exists agreement between experimental and model simulations.

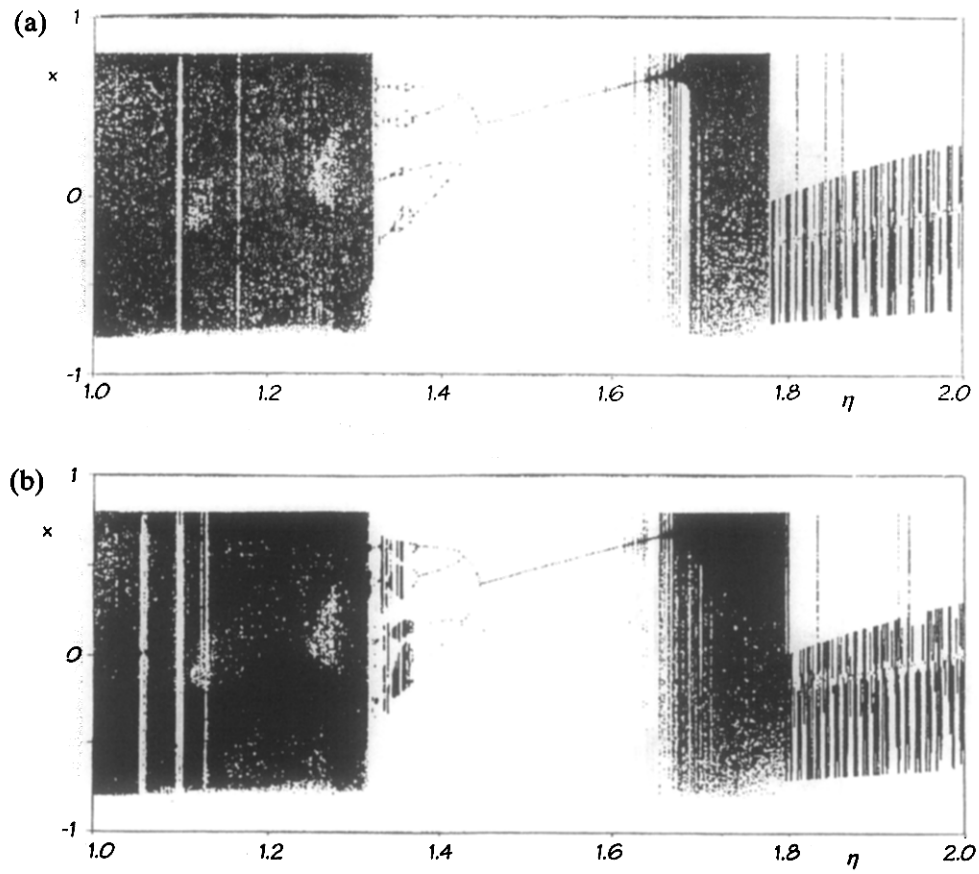

Fig. 4. Bifurcation diagram of system (1) $r = 0.8$, $\delta = 0.5$, $b_1 = 0.1$, $\mu = 0.693$ and $R = 0.62$, (a) linear friction force (4): $\lambda = 0.02$, (b) nonlinear friction force (5): $\mu_0 = 0.25$, $\mu_1 = 0.05$, $\lambda_1 = 1.42$ and $\lambda_2 = 0.005$.

4. CONCLUSIONS

Our investigations confirm that a rich bifurcation structure and chaotic behaviour are typical for mechanical systems with dry friction and impacts. The analysis of bifurcation diagrams shows strong sensitivity of qualitative behaviour on the small changes of system parameters such as the friction and restitution coefficients. As both these parameters are estimated experimentally with an error of a few per cent [20], our observations show that it will be very difficult or even impossible to obtain good agreement between experimental and numerical results for all system parameters. Despite this sensitivity, the sequence of bifurcations seems to be independent of the friction and restitution coefficients as well as of the model of dry friction.

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