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Synchronization of chaotic oscillators by periodic parametric perturbations

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Abstract

We show that the synchronization of coupled chaotic oscillators can be achieved by means of periodic parametric perturbations of the coupling element. The possibility of synchronization is demonstrated for the simple case of two identical nonautonomous oscillators with piecewise linear characteristics both analytically and numerically. Using linear analysis we have determined the stability conditions for symmetric oscillations.

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Recently, the problem of synchoronization and controlling chaos in different dynamical systems has attracted great interest. It has been established that two interacting nonlinear systems can demonstrate the phenomenon of synchronization of chaos for some coupling [1–7]. The synchronous regime of chaotic dynamical systems can be achieved also through a controlling chaos procedure based on different methods [8–14]. In this work we have studied the possibility of synchronization of coupled identical chaotic oscillators by means of the periodic parameter perturbations of the coupling element.

The application of periodic parametric perturbations for modification of the chaotic dynamics has been considered in [15–20], where it has been shown both theoretically and experimentally that reasonant parametric perturbations can suppress chaotic behaviour.

The idea of using the parametric perturbations for synchronization of coupled chaotic oscillators is based on a familar classical problem of the pendulum with oscillated suspension (the suspension point of the pendulum moves harmonically) [21,22]. Beginning from the defined values of amplitude and frequency the parametric perturbation change the unstable equilibrium state into the stable one.

In this work, for the simple example of two coupled identical nonautonomous chaotic oscillators we show the stabilization effect of symmetrical motions (that correspond to synchronization of chaos) by means of the periodic parametric perturbations of the coupling element. The stability condition of symmetrical synchronized oscillations is found and the region of the synchronization of chaos in the parameter space (the amplitude and the frequency of parametric perturbations) is located. The numerical results have shown good agreement with theoretical ones.

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Let us consider two coupled identical oscillators in the form

$$\ddot{x}_{1,2} + \alpha\dot{x}_{1,2} + f(x_{1,2}) - \gamma(x_{2,1} - x_{1,2}) = B\sin(\omega t), \tag{1}$$

where

$$f(x) = (b-1)x + 0.5(b-a)(|x-1| - |x+1|), \tag{2}$$

α is the dissipation parameter, γ the coefficient of coupling, and B and ω are the amplitude and the frequency of the external force, respectively.

In case $\gamma = 0$, Eqs. (1) are uncoupled and when the external force is equal to zero ($B = 0$), Eqs. (1) describe the nonlinear damping oscillator with three equilibrium points:

$$\begin{aligned} &P_1: \quad (x_1 = 0,\ y_1 = 0),\\ &P_2: \quad (x_2 = (b-a)/(b-1),\ y_2 = 0),\\ &P_3: \quad (x_3 = -(b-a)/(b-1),\ y_3 = 0). \end{aligned}$$

If

$$0 < a < 1, \quad b > (1 + \tfrac{1}{4}\alpha^2), \quad \alpha > 0,$$

the point P_1 is the saddle one and $P_{2,3}$ are the stable focus points. The external force at which the nonlinear oscillator shows the chaotic behaviour in the region of the parameter values has been estimated. The coupled oscillators ($\gamma > 0$) show different forms of the regular and chaotic behaviour including the regime of the nonsynchronous chaotic oscillations.

In the system (1) the synchronization of chaotic oscillators can be achieved by means of periodic parametric perturbations of the coupling coefficient. Suppose that the coupling coefficient γ is varied periodically in time about the constant level γ_0, i.e.,

$$\gamma = \gamma_0 + F(t), \tag{3}$$

where $F(t)$ is the periodic function with the period $T = 2\pi/\Omega$ (Ω is the frequency of parametric perturbations). In order to demonstrate the mechanism of the stabilization of the symmetric oscillations more simply, we will take the parametric perturbation in the form

$$F(t) = \epsilon\Omega^2\,\mathrm{sgn}(\sin(\Omega t)). \tag{4}$$

Taking into account (3) and (4), we can rewrite the equation of the coupled oscillators (1) as

$$\begin{aligned} \dot{x}_{1,2} &= y_{1,2},\\ \dot{y}_{1,2} &= -\alpha y_{1,2} - f(x_{1,2}) + (\gamma_0 + \epsilon\Omega^2\,\mathrm{sgn}(\sin(\Omega t))(x_{2,1} - x_{1,2}) + B\sin(\omega t). \end{aligned} \tag{5}$$

With the variable transformations

$$u = \tfrac{1}{2}(x_1 - x_2), \qquad u_1 = \tfrac{1}{2}(x_1 + x_2), \qquad v = \tfrac{1}{2}(y_1 - y_2), \quad v_1 = \tfrac{1}{2}(y_1 + y_2),$$

we can get from the system (5) equations which describe dynamics of the coupled oscillators in a small vicinity of a symmetric subspace of the complete phase space

$$\dot{u} = v, \tag{6}$$

$$\dot{v} = -\alpha v - \omega_0^2(t)u, \tag{7}$$

$$\dot{u}_1 = v_1, \tag{8}$$

$$(\dot{v})_1 = -\alpha v_1 - f(u_1) + B\sin(\omega t), \tag{9}$$

where

$$\omega_0^2(t) = A + 2\gamma_0 + 2\epsilon\Omega^2 \operatorname{sgn}(\sin(\Omega t)), \qquad f(u_1) = \tfrac{1}{2}(f(x_1) + f(x_2)).$$

$$A = \begin{cases} a-1 & \text{if } |x| \le 1, \\ b-1 & \text{if } |x| > 1. \end{cases}$$

Eqs. (8) and (9) describe the system dynamics in the symmetric synchronization subspace ($x_1 = x_2$, $y_1 = y_2$) of the complete phase space. Eqs. (6) and (7) describe evolution transverse to the synchronization subspace, so the values $u = 0$, $v = 0$ correspond to the synchronous regime. In order to define the conditions of the stability of the synchronized chaotic oscillations we investigate the evolution of small perturbations over the parametric influence period ($T = 2\pi/\Omega$) of the fixed point $u = 0$, $v = 0$ (Eqs. (6) and (7)). It can be easily carried out by using a familiar technique (see, e.g., [23]). After some algebbraic manipulations one can find the eigenvalues $\mu_{1,2}$ which characterize the stability of the symmetric oscillations

$$\mu_{1,2} = \exp(-\alpha\pi/\Omega)(0.5S \pm (0.5S^2 - 1)^{1/2}), \tag{10}$$

where

$$S = 2(Ch(\beta_1)Ch(\beta_2) + KSh(\beta_1)Sh(\beta_2)),$$
$$\beta_1 = (\pi/\Omega)(0.25\alpha^2 - A - 2\gamma_0 - e\epsilon\Omega^2)^{1/2},$$
$$\beta_2 = (\pi/\Omega)(0.25\alpha^2 - A - 2\gamma_0 + 2\epsilon\Omega^2)^{1/2},$$
$$K = (0.25\alpha^2 - A - 2\gamma_0)/((0.25\alpha^2 - A - 2\gamma_0)^2 - 4\epsilon^2\Omega^4)^{1/2}.$$

Synchronization was observed, when $|\mu_{1,2}| < 1$ at either values of x from the vicinity of the symmetric phase subspace, i.e., both at $A = \alpha - 1$ and at $A = b - 1$. It should be noted here that the general conclusion about the stability of the synchronized state for $|\mu_{1,2}| < 1$ is not straightforward as the considered system is piecewise linear and the conclusion about its stability cannot be made from the stability of the system in each linear part of the phase space.

On the parameter plane of Fig. 1 we have plotted the region where the stabilization condition of the symmetric motions is satisfied. The values of other parameters of the system of coupled oscillators ($B = 1.5$, $\omega = 1$, $\alpha = 0.1$, $\gamma_0 = 0.1$, $a = 0.5$, $b = 2$) are consistent with the case when the regime of nonsynchronous chaotic oscillations is observed in the absence of the parametric perturbation ($\epsilon = 0$).

The computer experiments on the system (5) have verified the following. The synchronization of chaotic oscillators can be achieved by means of periodic parametric perturbations of the coupling element. The synchronization of chaos is observed namely at such values of the amplitude and the frequency of parametric perturbations which are shaded on the parameter plane of Fig. 1. As an illustration, the projections of the phase trajectories are presented in Fig. 2 in the absence of the parametric perturbations (Fig. 2(a)) and with one (Fig. 2(b)). The numerical simulation has been carried out as follows. At above the parameter values, the initial conditions are chosen from the small vicinity of the symmetric subspace: $x_1(0) = x_2(0) + \Delta x$, $y_1(0) = y_2(0) + \Delta y$. Usually, we choose $\Delta x = 0.02$, $\Delta y = 0.02$. Then the phase portraits and the time-series of the oscillation regimes are plotted at the varied values of the amplitude ϵ and the frequency Ω. At $\epsilon = 0$ the phase trajectory leaves the vicinity of the symmetric subspace and the nonsynchronous chaotic oscillations are observed (see Fig. 2(a)). With parametric perturbations the initial deviations Δx and Δy decrease. The phase trajectory enters into the symmetric subspace and evolves therein, if the

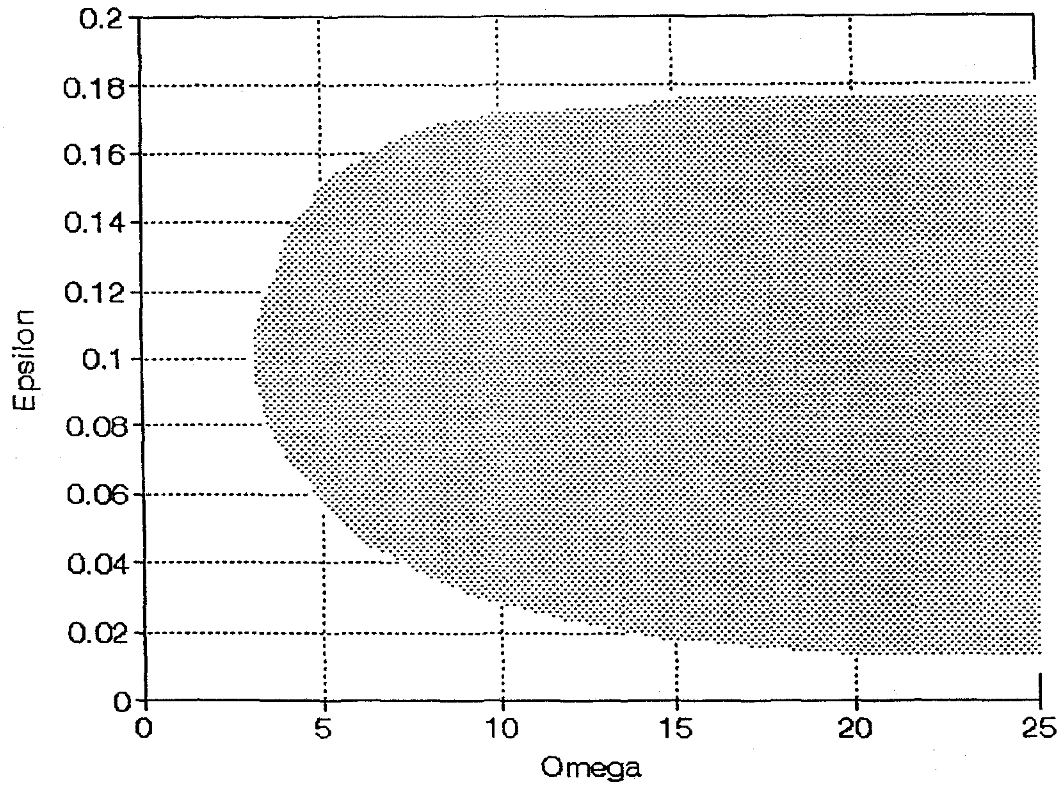

Fig. 1. Region of symmetric oscillation stability on the parameter plane (ϵ, Ω). The values of other parameters are fixed at $B = 1.5$, $\omega = 1$, $\alpha = 0.1$, $\gamma_0 = 0.1$, $a = 0.5$, $b = 2$.

values ϵ and Ω are among the dashed region on the parameter plane of Fig. 1. If the values are chosen outside this region the initial deviations $(\Delta x, \Delta y)$ increase and the synchronous regime is not achieved.

As can be seen in Fig. 1 the frequency of the parametric perturbations, by which the synchronization of chaos is achieved, has to be larger than the value of the characteristic frequency of the nonperturbed motion (in the considered case it is the frequency $\omega = 1$). In order to achieve the synchronous regime in the system (5) it is sufficient that the amplitude of the perturbation of the coupling coefficient accounts to about 20% of $\gamma_0 = 0.1$, the frequency of which is 15–20 times as $\omega_0 = 1$ (see Fig. 1).

The preliminary numerical investigations have shown that the synchronization of chaos appears also in the case, when the phase point is placed far from the symmetric subspace at the moment of the application of the parametrical perturbation. However, in this case, the time of the transition process to the synchronous chaotic regime increases substantially. In the perturbed system the time of reaching of the phase symmetric subspace vicinity, where the stabilization mechanism will have acted, is substantially greater than that in the nonperturbed system. In conclusion, we have demonstrated the possibility of synchronization of coupled chaotic oscillators by means of periodic parametric perturbations of the coupling element.

It should be noted here that dynamical systems with invariant manifolds are not generic. In most of the synchronization schemes of two identical systems the invariant manifold is a consequence of the symmetry in the coupled system. The effect of a small asymmetry due to the small differences in the considered systems on the stability of the synchronized state was studied elsewhere [24].

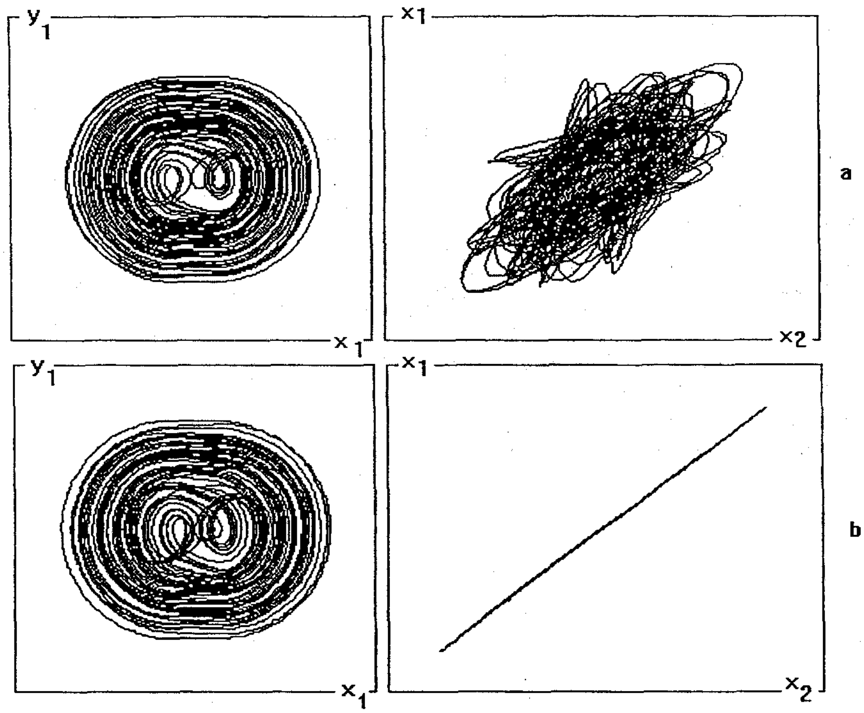

Fig. 2. Projection of the phase trajectories in the absence (a) and in presence (b) of the parametric perturbations. Value of parameters are $B = 1.5, \omega = 1, \alpha = 0.1, \gamma_0 = 0.1, a = 0.5, b = 2$. ((b) can be obtained at any ϵ and Ω from the dashed region on the parameter plane in Fig. 1).

We have also mentioned that the presented method can be considered as one of the controlling chaos methods. Chaotic attractor, according to the nonsynchronous oscillations, contains the chaotic subset which lies in the symmetric subspace of the complete phase space. When the dynamical system evolves on the chaotic attractor, its phase point visits from time to time the small vicinity of the symmetric subspace. If the periodic parametric perturbation is applied at this moment, the phase point enters the symmetric subspace and evolves therein and the parametric perturbation does not act on the regime of symmetric motions.

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