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Riddling bifurcations in coupled piecewise linear maps

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Abstract

A mechanism for riddling bifurcations in the system of coupled piecewise linear maps is described. We give sufficient conditions for the occurrence of locally and globally riddled basins based on the properties of absorbing areas of the chaotic attractors on the invariant manifold. It is also shown that riddled basins are preserved upon bifurcation of the chaotic attractors as long as the attractor after bifurcation is located in the absorbing area of the attractor before bifurcation. ©1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Recently, riddled basins [1–5] have been found in a great number of systems and become an important study area in nonlinear dynamics. Riddling typically occurs in chaotic systems with invariant manifolds, as for example in chaos synchronization schemes [6–18]. In such systems riddling can lead either to the temporal (local riddling) or permanent (global riddling) loss of synchronization.

Consider two identical chaotic systems, $x_{n+1} = f(x_n)$ and $y_{n+1} = f(y_n)$, $x, y \in \mathbb{R}$, evolving on an *asymptotically stable attractor* A (the set A is an asymptotically stable attractor if for any sufficiently small neighborhood $U(A)$ of A there exists a neighborhood $V(A)$ of A such that if $x \in V(A)$, then $f^n(x) \in U(A)$ for any $n \in \mathcal{Z}^+$ and distance $\rho(f^n(x); A) \to 0, n \to \infty$). With one-to-one coupling the map becomes a two-dimensional map $f(x, y) : \mathbb{R}^2 \to \mathbb{R}^2$, given by equations

$$\begin{aligned} x_{n+1} &= f(x_n) + d_1(y_n - x_n), \\ y_{n+1} &= f(y_n) + d_2(x_n - y_n). \end{aligned} \tag{1}$$

In this case the system can be synchronized for some ranges of $d_{1,2} \in \mathbb{R}$, i.e., $| x_n - y_n | \to 0$ as $n \to \infty$ and its limiting dynamics is restricted to a one-dimensional invariant subspace $x_n = y_n$ [1,6].

We define the basin of attraction $\beta(A)$ to the set of points whose ω-limit set is contained in A. In Milnor's definition [19] of an attractor, the basin of attraction need not include the whole neighborhood of the attractor, we say that A is a weak Milnor attractor if $\beta(A)$ has positive Lebesgue measure. If the basin $\beta(A)$ is riddled, it has positive Lebesgue measure but does not contain any neighborhood of the attractor A

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(A is a weak Milnor attractor [19]). In this case the basin of attraction $\beta(A)$ may be a fat fractal so that any neighborhood of the attractor intersects the basin with positive measure, but may also intersect the basin of another attractor with positive measure.

The dynamics of the system (1) is described by two Lyapunov exponents. One of them describes the evolution on the invariant manifold $x = y$ and is always positive. The second exponent characterizes evolution transverse to this manifold and it is called transversal. If the transversal Lyapunov exponent, $\lambda_\perp$ is negative, the set A is an attractor, at least in the *weak Milnor sense* [4], i.e., the set A is a *weak Milnor attractor* if its basin of attraction $\beta(A)$ has positive Lebesgue measure in $\mathbb{R}^2$.

When the transversal Lyapunov exponent is negative, but there exist trajectories in the attractor A which are transversally repelling, A is a weak Milnor attractor with a *locally riddled* basin (a set A is an attractor with a locally riddled basin of attraction, if there is a neighborhood U of A such that given any neighborhood V of any point in A, there is a set of points in $V \cap U$ of positive Lebesgue measure which leaves U in a finite time).

This riddling property has a local character. It describes behavior only in a sufficiently small neighborhood $U = U(A)$ and it gives no information about further behavior of the trajectories after they leave U. Two different classes of riddled basins have been defined [29,33]. In the case of a *locally riddled basin* almost all (in the sense of measure) trajectories leaving the neighborhood $U(A)$, come back to U. Then, some fraction of them, after a finite number of iterations, leave U again, and so on. The dynamics of such trajectories displays non-regular, temporal "bursting": a trajectory spends some time (usually long) near attractor A until it goes away; then, after some other time (usually short) it re-enters the neighborhood U. If the basin of attractor A is a *globally riddled basin*, a positive measure set of points after leaving $U(A)$, goes to another attractor (or attractors) or to infinity.

Only in the globally riddled case can the basin of attraction $\beta(A)$ have the riddled structure of a fat fractal as a subset of $\mathbb{R}^2$. In this case the neighborhood of any point $(\bar{x}, \bar{y}) \in \beta(A)$ is filled by a positive Lebesgue measure set of points (x, y) which are attracted to another attractor (attractors).

In this paper we identify and describe the special features of bifurcations leading to locally and globally riddled basins in the system of coupled, piecewise linear maps. Special attention is given to the differences between these bifurcations and equivalent bifurcations in coupled, smooth systems. Additionally, we consider the influence of the bifurcation of a chaotic attractor A on the structure of its basin of attraction.

The outline of this paper is as follows. In Section 2 we describe the system under consideration and recall its fundamental properties. Two different bifurcation scenarios leading to globally riddled basins are identified and studied in Section 3. Section 4 investigates the influence of the bifurcations of the attractors located in the invariant manifold on the structure of its basins of attraction. Finally, we summarize our results in Section 5.

2. A model

In this paper we identify and describe riddling bifurcations of a four-parameter family of two-dimensional, piecewise linear, non-invertible map:

$$\begin{aligned} F &= f_{l,p}(x_n) + d(y_n - x_n), \\ x_{n+1} &= px_n + \frac{l}{2}\left(1 - \frac{p}{l}\right)\left(\left|x_n + \frac{1}{l}\right| - \left|x_n - \frac{1}{l}\right|\right) \\ &\quad + d_1(y_n - x_n), \end{aligned}$$

$$\begin{aligned} F &= f_{l,p}(y_n) + d(x_n - y_n), \\ y_{n+1} &= py_n + \frac{l}{2}\left(1 - \frac{p}{l}\right)\left(\left|y_n + \frac{1}{l}\right| - \left|y_n - \frac{1}{l}\right|\right) \\ &\quad + d_2(x_n - y_n), \end{aligned} \tag{2}$$

where $l, p, d_{1,2} \in \mathbb{R}$. Note that this system, which consists of two identical linearly coupled one-dimensional maps, is the generalization of the skew tent map. Chaotic attractors of skew tent maps have been considered in [23–27]. The two-dimensional map F, given by Eq. (1), is non-invertible as long as the one-dimensional map f is non-invertible in the sense that there exist regions (points) having two or four preimages. To characterize the properties of system (1) we need some notation from the theory of non-invertible maps (for details see [30,34]). If A is a connected attractor, the *immediate basin* $\bar{\beta}(A)$ is defined as the widest connected component of $\beta(A)$ containing A. Inside the immediate basin $\bar{\beta}(A)$ one can define an *absorbing area*. An area $\mathcal{A}(A)$ is said to be absorbing if (i) $F(\mathcal{A}(A)) = \mathcal{A}(A)$, i.e., it is invariant with respect to F, (ii) $\mathcal{A}(A)$ is attracting in the following strong sense: there exists a neighborhood $U(\mathcal{A}(A))$ such that all its points are mapped inside $\mathcal{A}$ in a finite number of iterations. The boundary $\delta\mathcal{A}(A)$ is made up of segments of critical curves. Critical curves l_k, $k = 0, 1, \ldots$, are defined as successive images $l_k = F^k(l_0)$, $k = 1, 2, \ldots$, of the curve (curves) l_0 which plays the same role in two-dimensional maps as a critical (i.e., extremum) point (points) for one-dimensional maps. l_0 can be defined as a curve of merging preimages of F [30]. It is clear that l_0 belongs to the set $\{(x, y)$: Jacobian of F, i.e., $|DF|$ vanishes or does not exist$\}$. For the considered map F, l_0 consists of two horizontal and two vertical lines: $l_0 = \{x = \pm(1/l), y \in \mathbb{R}\} \cup \{x \in \mathbb{R}, y = \pm(1/l)\}$. It should be mentioned here that the concept of absorbing areas is related to the idea of essential basins [36] restricted to compact sets.

We now return to the two-dimensional map of the plane (x, y) into itself given by Eq. (2). When $(l, p) \in \Pi = \{l > 1, -1/(l-1) < p \leq -1\}$ the one-dimensional map $f_{l,p}$ has two symmetrical attractors $\Gamma^{(-)} \subset [-1, 0]$ and $\Gamma^{(+)} \subset [0, 1]$, which are cycle of 2^m chaotic intervals (the so-called 2^m-piece chaotic attractors). Depending on the parameters l and p, m can be any positive integer. Denoting Π_m to be a subregion of Π where $\Gamma_m^{(\pm)}$ is a period-2^m cycle of chaotic intervals, the bifurcation curves for the transition $\Gamma_m^{(\pm)} \to \Gamma_{m+1}^{(\pm)}$ form the boundary between Π_m and Π_{m+1}.

For the map $F_{l,p}$, each set $A = A_m^{(\pm)} = \{x = y \in \Gamma_m^{(\pm)}\}$ is a one-dimensional chaotic invariant set which may or may not be an attractor in the plane (x, y). The various notions of attractors involve two kinds of properties: (i) that it attracts nearby trajectories and (ii) that it cannot be decomposed into smaller attractors. We shall concentrate on the first property, since A is filled with dense trajectories and 'good' invariant SBR measure μ which is absolutely continuous with respect to Lebesgue measure [35]. Thus, the definitions given here should be completed by some minimality condition in order to be generally valid.

In comparison with the maps studied in [1–5] our map (2) has the advantage that, as shown in [28,29], conditions for the occurrence of locally riddled basins and conditions for the attractor A to be asymptotically stable can be given analytically, it is therefore a useful test model for coupled chaotic systems.

3. Necessary conditions for riddling bifurcations

Riddling bifurcation occurs when some point x_p (unstable fixed or periodic point embedded in the chaotic attractor A in the invariant manifold) loses its transverse stability as a coupling parameter p passes through the critical value $p = p_c$ [20–22]. Lai et al. [22] showed that the loss of transverse stability may be induced by the collision at $p = p_c$ of two repellers r_+ and r_-, located symmetrically with respect to the invariant manifold, with the saddle at x_p. As the result of this bifurcation a tongue opens at x_p and all preimages of x_p, allowing trajectories near the invariant manifold to escape from the U neighborhood of A for $p > p_c$. As preimages of x_p are dense in the invariant manifold there is an infinite number of tongues, and a set of points leaving the U neighborhood has a positive Lebesgue measure. Note that this scenario is for the case when x_p undergoes (in transversal direction) a subcritical pitchfork bifurcation (corresponding multiplicator crosses through $+1$). A similar riddling mechanism works when x_p undergoes a subcritical period-doubling bifurcation (multiplicator crossing through -1). In this case the loss of transversal stability is induced by the collision at $p = p_c$ of the only one repeller r of the double period, points of which are located symmetrically with respect to the invariant manifold, with the saddle at x_p. Different riddling scenarios occur in the case when saddle x_p undergoes a supercritical bifurcation in the transversal direction [31–33].

We have found here that the riddling scenarios for the piecewise linear map (2) are quite different: at the moment when the transversal multiplicator of the saddle at x_p crosses $+1$ (or -1) there is not a direct collision of the repellers (or one repeller) with the saddle. Before the bifurcation, these repellers (or one repeller) can exist but are located far from the invariant manifold. The bifurcation can change each of them into a saddle, or even into the attractor [23,25].

After leaving U a typical trajectory may or may not be captured by another attractor. If it is not captured it returns to U and the basin of attractor A is *locally riddled*. We will shortly give evidence that in order for a trajectory of the system (2) leaving the neighborhood of U to be captured by another attractor, one of the following conditions has to be fulfilled:

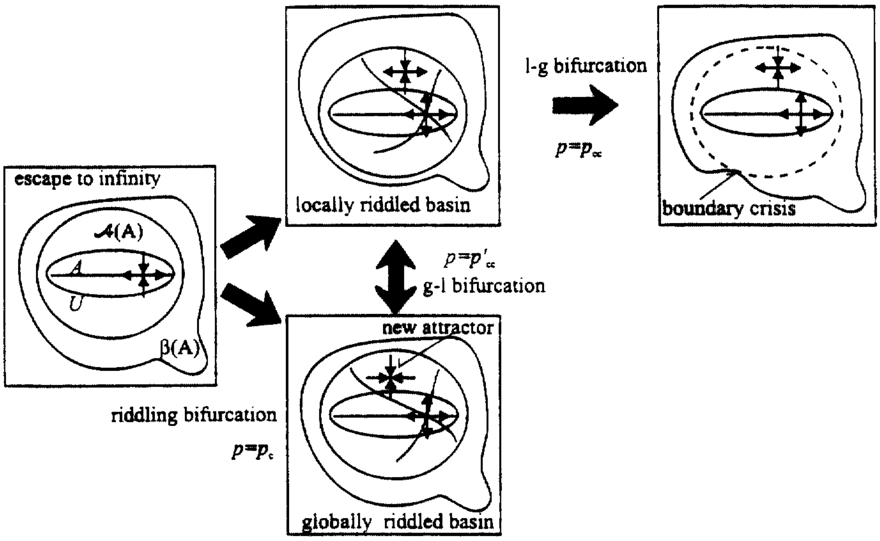

Fig. 1. Scenarios of riddling bifurcations in piecewise linear systems.

(i) The boundary of the immediate basin $\tilde{\beta}(A)$ is broken as the result of the boundary crisis with the absorbing area $\mathcal{A}(A)$,

(ii) A new attractor has to be born inside the absorbing area $\mathcal{A}(A)$ in the moment of riddling, i.e., at $p = p_c$.

In the first case after a riddling bifurcation in which a point x_p loses its transverse stability we observe transition to locally riddled basins. Globally riddled basins can be established after boundary crisis of absorbing area $\mathcal{A}(A)$ and the immediate basin of attraction $\tilde{\beta}(A)$. This global bifurcation, which occurs at $p = p_{cc}$, we call a l–g *(local–global) riddling bifurcation* and this transition to globally riddled basins is called the outer one.

In the second scenario the globally riddled basins are established as the result of a riddling bifurcation which in this case gives rise to the birth of the new attractor (attractors). This transition will be called the inner one as it occurs inside the basin $\tilde{\beta}(A)$. The new attractor (attractors) born as the result of this bifurcation will later lose its stability due to the boundary crisis with its basin boundary, and we observe the transition from globally to locally riddled basins (g–l *bifurcation*). Further variations of the parameter d can result in l–g bifurcation as in the previous scenario.

The bifurcations leading from an asymptotically stable attractor A to a globally riddled attractor are shown in Fig. 1.

The described inner transition is characteristic for piecewise linear maps. In the case of smooth maps the transition from asymptotic stability of the synchronized chaotic attractors to globally riddled basins leads through the locally riddled regime [33].

Results of [28,29] guide the choice of parameter values, and we can illustrate two types of riddling bifurcations in Figs. 2 and 3 for $l = -p = 1.55$.

An example of an inner type of l–g bifurcation is shown in Fig. 2(a)–(c). Before the bifurcation (Fig. 2(a), $d_1 = d_2 = -0.92$), both attractors $A^{(+)}$ and $A^{(-)}$ are asymptotically stable. At the bifurcation, $d_1 = d_2 = -0.923$, two new attractors $A^{(+)}_{1,2}$ and $A^{(-)}_{1,2}$ are born inside the absorbing areas of the attractors $A^{(+)}$ and $A^{(-)}$ and the basins of these attractors become globally riddled as can be seen in Fig. 2(b) ($d_1 = d_2 = -0.925$) and at the enlargement in Fig. 2(c). At $d_1 = d_2 = -0.95$, attractors $A^{(+)}_{1,2}$ and $A^{(-)}_{1,2}$ become unstable (they are transformed into chaotic saddles) due to the boundary crisis with its basin boundary shown in Fig. 2(d) ($d_1 = d_2 = -0.948$) and we observe transition from globally to locally riddled basins (g–l *bifurcation*). In the locally riddled regime chaotic saddles exist inside absorbing areas $\mathcal{A}(A^{(+)})$ and $\mathcal{A}(A^{(-)})$. An

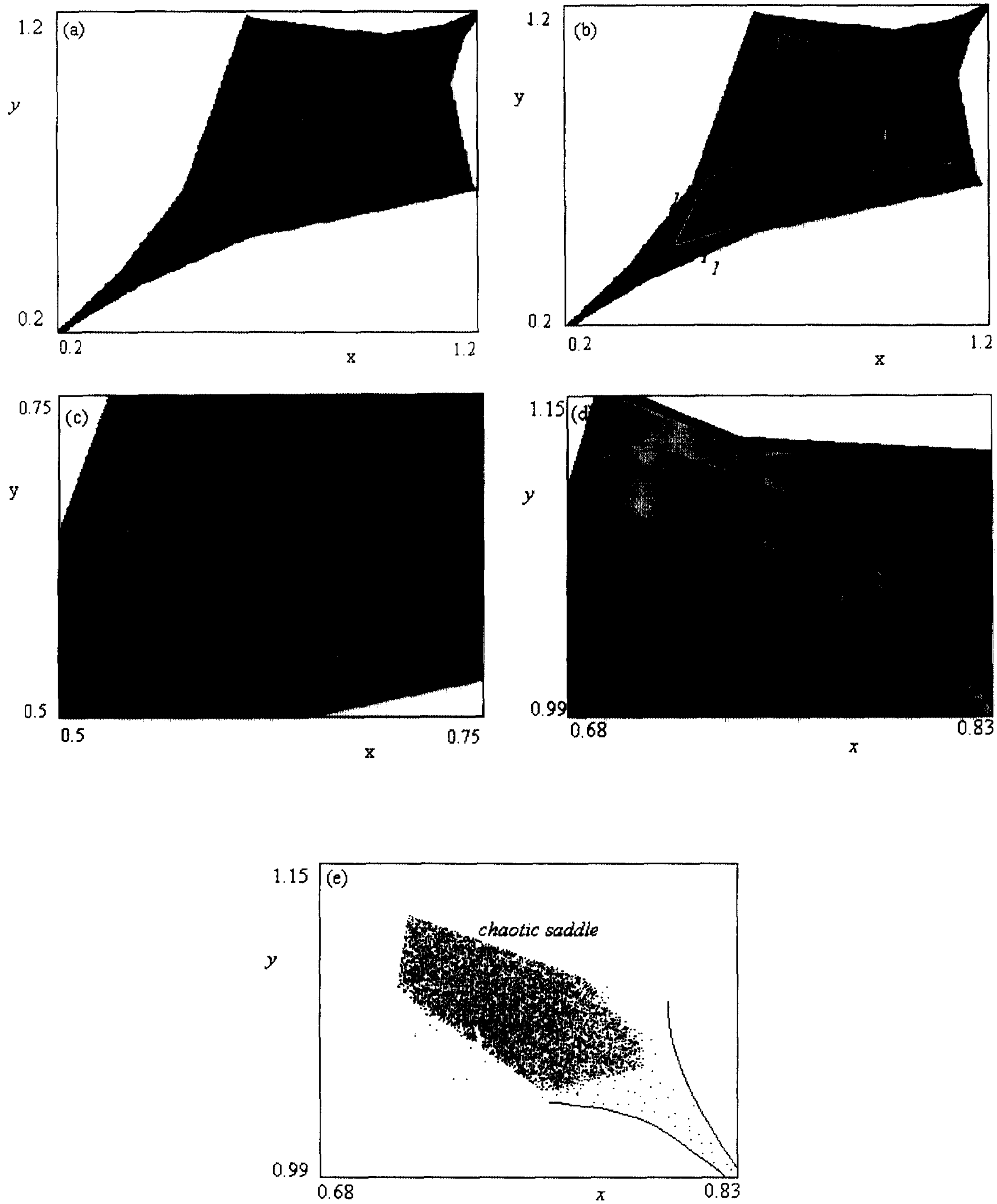

Fig. 2. Inner l–g riddling bifurcations of map (2); $l = 1.55$, $p = -1.55$: (a) $d_1 = d_2 = -0.92$, attractor $A^{(+)}$ is asymptotically stable; (b) $d_1 = d_2 = -0.925$, attractor $A^{(+)}$ is globally riddled by newly born attractors $A^{(+)}_{1,2}$; (c) enlargement of globally riddled basin for $d_1 = d_2 = -0.94$; (d) a moment shortly below g–l bifurcation, $d_1 = d_2 = -0.948$; (e) chaotic saddle for $d_1 = d_2 = -0.95$.

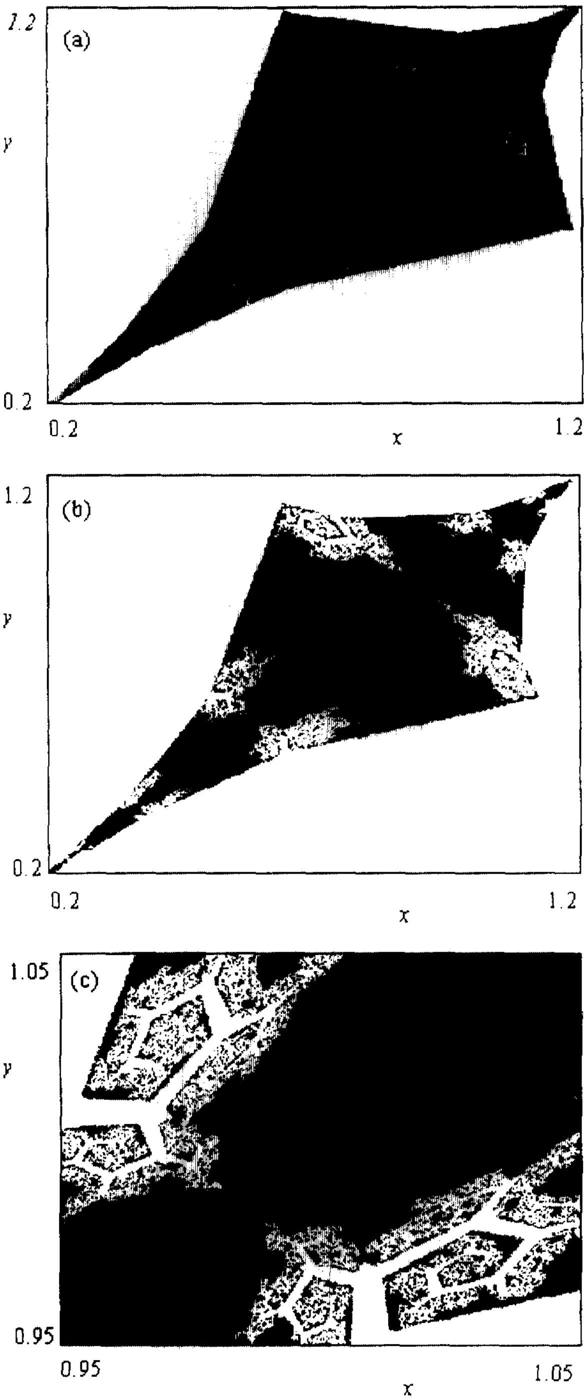

Fig. 3. Outer l–g riddling bifurcations of map 2; $l = 1.55, p = -1.55$: (a) $d_1 = d_2 = -0.965$, attractor $A^{(+)}$ is locally riddled: and (b), (c) $d_1 = d_2 = -0.980$, attractor A^+ is globally riddled by an attractor at infinity.

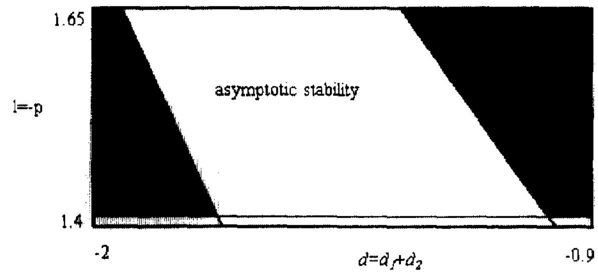

Fig. 4. Boundaries of riddling bifurcations in $l = -p$ versus $d = d_1 + d_2$ plane; black and grey areas indicate, respectively, globally and locally riddled basins.

example of these non-attracting chaotic sets calculated by the proper interior maximum (PIM) triple procedure [37] is shown in Fig. 2(e). System trajectories are leaving the neighborhoods of attractors $A^{(+)}$ and $A^{(-)}$ through the mushroom-shaped phase-space regions to spend some time on the chaotic saddles and then they are returning towards attractors $A^{(+)}$ and $A^{(-)}$.

In Fig. 3(a) and (b) we show one example of outer l–g bifurcation for map (2). Before the bifurcation in Fig. 3(a) ($d_1 = d_2 = -0.965$), the basins of attractors $A^{(+)}$ and $A^{(-)}$ are locally riddled basins (for this case the chaotic saddles similar to that of Fig. 2(e) can be found inside the absorbing areas). The boundary crisis between immediate basin of attraction $\bar{\beta}(A^{(+)})$ and absorbing area $\mathcal{A}(A^+)$ is indicated in Fig. 3(a). After a bifurcation in Fig. 3(b) and (c) ($d_1 = d_2 = -0.98$), these basins of attractors $A^{(+)}$ and $A^{(-)}$ are globally riddled by the attractor at infinity.

In both types of bifurcations leading to globally riddled basins the important role is played by the absorbing areas $\mathcal{A}(A^{(+)})$ and $\mathcal{A}(A^{(-)})$. As it was shown in Section 2, this area can be estimated numerically by iteration of lines l_0. Fig. 4 shows the boundaries of riddling bifurcations in $l = -p$ versus $d = d_1 + d_2$ plane. The boundaries between the areas of local and global riddling have been calculated either from the condition of boundary crisis of $\mathcal{A}(A^{(+)})$ and $\mathcal{A}(A^{(-)})$ with immediate basin boundary $\bar{\beta}(A^{(+)})$ and $\bar{\beta}(A^{(-)})$ (numerically the lines l_1, l_2, l_3, and l_4 are checked to see if they are attracted to $A^{(+)}$ and $A^{(-)}$) or by the birth of new attractor condition (numerically the area between lines l_1, l_2, l_3, and l_4 is checked to see if it is attracted to $A^{(+)}$ and $A^{(-)}$). Region of asymptotic stability was calculated from the analytical conditions

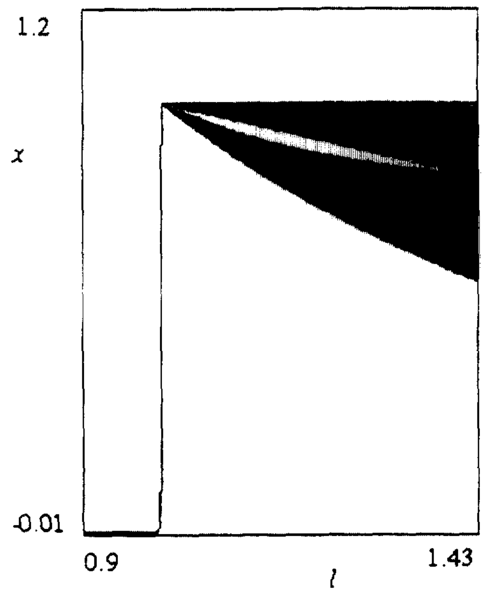

Fig. 5. Bifurcation diagram of the attractor $A^{(+)}$: $p = -\sqrt{2}$, $d_1 = d_2 = -0.935$.

given in [29]. Riddling bifurcations were found to be robust in the considered parameter space.

4. Influence of the bifurcations of chaotic attractor located at the invariant manifold on the structure of basins of attraction

Chaotic attractors located at the invariant manifold $x = y$ can undergo period-doubling bifurcations giving transition from 2^k-piece chaotic attractor to 2^{k+1}-piece chaotic attractor. In this section we analyze the influence of the bifurcations of attractors $A^{(+)}$ and $A^{(-)}$ on the structure of their basins of attraction.

In Fig. 5 we present the bifurcation diagram of the $A^{(+)}$ for $p = -\sqrt{2}, d_1 = d_2 = -0.935$, where l is taken as the bifurcation parameter. At $l = \sqrt{2}$ attractor $A^{(+)}$ bifurcates from one-piece to two-piece chaotic attractor. With further decrease of l the two-piece chaotic attractor becomes smaller and smaller and finally is replaced by the $x = 0$ fixed point attractor (attractor $A^{(-)}$ undergoes the same bifurcations).

The structure of globally riddled basins of attraction of attractors $A^{(+)}$ and $A^{(-)}$ is preseved as long as these attractors are chaotic as can be seen in Fig. 6(a)–(c). In Fig. 6(a) we present the globally riddled basin of attraction of the one-piece chaotic attractor $A^{(+)}(l = \sqrt{2})$. Similar structure is visible after the bifurcation to the two-piece chaotic attractor in Fig. 6(b) ($l = 1.2$) and (c) ($l = 1.006$) shortly before $A^{(+)}$ bifurcates to $x = 0$ attractor.

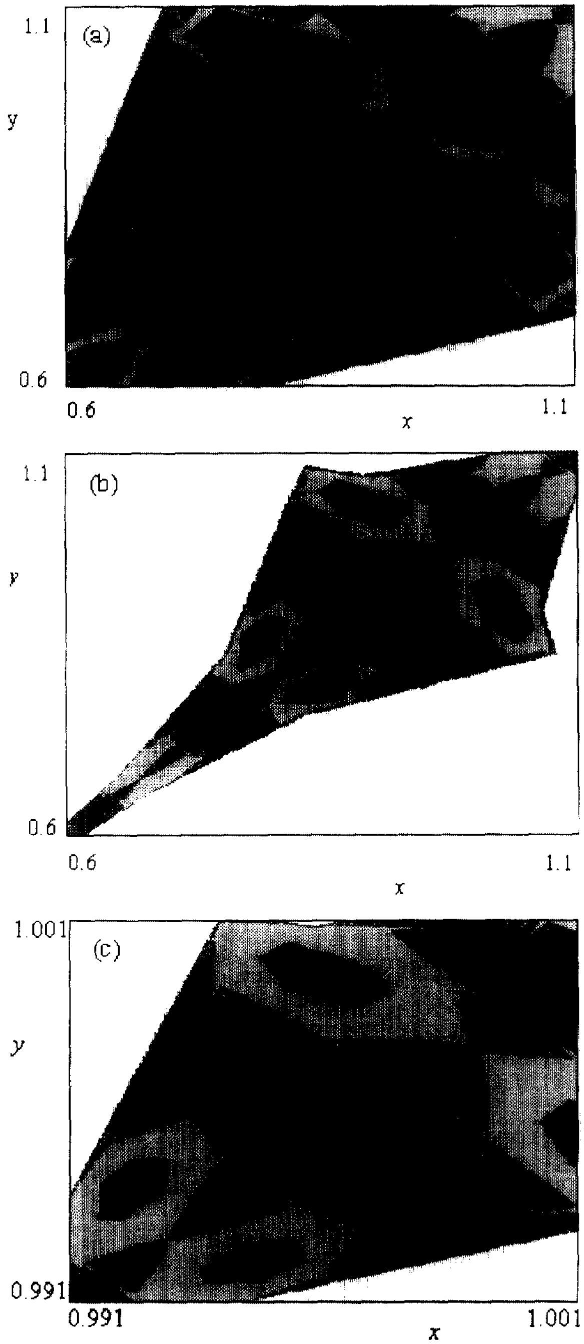

Fig. 6. Globally riddled basins of attractor $A^{(+)}$: $p = -\sqrt{2}$, $d_1 = d_2 = -0.935$; (a) $l = \sqrt{2}$, (b) $l = 1.200$, (c) $l = 1.006$.

Generally, our analysis shows that the structures of globally and locally riddled basins are preserved with bifurcations of $A^{(+)} \to A'^{(+)}$ and $A^{(-)} \to A'^{(-)}$ attractors if the attractors after bifurcation, $A'^{(+)}$ and $A'^{(-)}$, are located in the absorbing areas $\mathcal{A}(A^{(+)})$ and $\mathcal{A}(A^{(-)}$ of attractors $A^{(+)}$ and $A^{(-)}$ before the bifurcation.

5. Conclusions

In this paper we identified the global phenomena which lead to the creation of riddled basins in coupled piecewise linear systems. These might be either a breaking of an existing immediate basin boundary due to the boundary crisis with an absorbing area, permitting an escape to a distant attractor, or the creation of a new attractor(s) within the existing absorbing area in the moment of riddling bifurcation; the basin(s) of the new attractor(s) riddle the basin of the initial attractor.

In the second case we observed a direct transition from asymptotic stability of attractors in the invariant manifold to the global riddling of their basins. This scenario of the riddling bifurcation giving instant rise to the birth of the new attractor(s) inside the absorbing area is characteristic for piecewise linear maps.

Additionally, we showed that the globally (locally) riddled structure of basins of attraction of chaotic attractors located at the invariant manifold is preserved with bifurcations of these attractors as long as attractors which occur as the result of bifurcation are located inside the absorbing area of attractors before bifurcation.

We stress that the model system equation (2) was used only for the purpose of illustrating the fundamental mechanism of l–g bifurcation. The observed properties of l–g bifurcations seem to be typical for a class of systems with lower-dimensional invariant manifolds, and are important both for the understanding and control of chaos synchronization.

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