19 July 1999

PHYSICS LETTERS A

Physics Letters A 258 (1999) 99–102

www.elsevier.nl/locate/physleta

Non-bifurcational mechanism of loss of chaos synchronization in coupled non-identical systems

V. Astakhov [a], T. Kapitaniak [b,c], A. Shabunin [a], V. Anishchenko [a]

[a] *Physics Department, Saratov State University, Astrachanskaya 83, 410071 Saratov, Russia*
[b] *Division of Dynamics, Technical University of Lodz, Stefanowskiego 1/15, 90-924 Lodz, Poland*
[c] *Institute for Plasma Research, University of Maryland, College Park, MD 20742, USA*

Received 20 May 1998; received in revised form 24 February 1999; accepted 4 May 1999
Communicated by A.P. Fordy

Abstract

We show that in the weakly non-identical coupled systems, the loss of synchronization (the destruction of a chaotic attractor located in the vicinity of the invariant subspace of identical systems) can be initiated by the smooth shift of one of these orbits out of the chaotic attractor. © 1999 Elsevier Science B.V. All rights reserved.

PACS: 05.45.+b

It is well-known that coupled identical systems can demonstrate exactly the same chaotic evolution [1–9], i.e. all subsystems do exactly the same thing at the same time. This cooperative behavior is defined as one kind of the chaos synchronization [10–14] and is important in the studies of continuous systems with uniform movement, neurons models, and coupled lasers, and electrical circuits. In the synchronization regime the chaotic attractor of two systems (number of systems two was taken to simplify the notation, but the discussion is valid for any number of them) $x_{n+1} = f(x_n)$ and $y_{n+1} = f(y_n)$ coupled together is located in the symmetric subspace $x = y$ of the phase space of coupled systems. When the system exits from the synchronization region, the chaotic state loses its transversal stability and is destroyed in a blowout bifurcation. As a rule, the intermittency and the very complex structure of the attractor basin (the so-called, bubbling and riddling transitions) accompany the loss of stability of chaotic attractor located in symmetric subspace [15–22].

In the studies of coupled identical subsystems it has been found out that the loss of the chaotic synchronization is immediately connected with bifurcations of saddle periodic orbits embedded in the chaotic attractor [16,23–26]. For instance, in the work [25] it has been demonstrated that the loss of phase synchronization begins with a saddle-node bifurcation of the unstable periodic trajectory embedded in the chaotic attractor. As a result of it, a specific intermittency regime (Eyelet Intermittency [25]) appears. In the work [23] it has been found out that a subcritical pitchfork bifurcation of the saddle point embedded in the symmetric chaotic attractor induces riddling transition.

0375-9601/99/$ - see front matter © 1999 Elsevier Science B.V. All rights reserved.
PII: S0375-9601(99)00291-1

In the work [26] we have investigated the bifurcation mechanism of the loss of stability of synchronous chaotic motions in coupled logistic maps:

$$x_{n+1} = \lambda_1 - x_n^2 + \epsilon_1 \left(x_n^2 - y_n^2 \right),$$
$$y_{n+1} = \lambda_2 - y_n^2 + \epsilon_2 \left(y_n^2 - x_n^2 \right). \qquad (1)$$

$(x_n, y_n$ are dynamical variables, $\lambda_{1,2}$ are controlling parameters of partial systems, $\epsilon_{1,2}$ are coefficients of coupling.) It has been shown that in the symmetric case $(\lambda_1 = \lambda_2 = \lambda, \epsilon_1 = \epsilon_2 = \epsilon)$ when the system approaches the point of blowout bifurcation, the sequence of soft bifurcations of the certain family of saddle orbits $2^N C^0$ (where 2^N is the orbit period, $N = 0,1,2,\ldots)$ takes place. This family of saddle orbits forms the skeleton of the synchronized chaotic attractor A^0. The loss of stability of the symmetric chaotic set A^0 in the normal direction begins with the bifurcation of the saddle point C^0 which induces the bubbling transition in the system. The saddle periodic orbit resulting from the bifurcation of the point C^0, and its unstable manifolds bound the region near the symmetric subspace from which a trajectory cannot leave. The bifurcation of this saddle periodic orbit located outside the symmetric subspace induces the riddling transition in the system. Bifurcations of saddle orbits with higher periods develop riddling phenomenon and lead to blowout bifurcation.

As the identity of systems is the non-generic case (impossible to be implemented in practical systems), from the point of view of experimental systems it is very important to investigate the influence of non-identity of coupled systems on bifurcation mechanism of the destruction of synchronous chaotic motions. Some aspects linked with influence of the system asymmetry have been discussed in works [18, 23].

In this letter, we will show that when the coupled systems are different, even slightly, the mechanism of the loss of chaos synchronization which leads to the destruction of the chaotic attractor located in the vicinity of $x_n = y_n$ subspace can be significantly different from that identified in the case of ideal subsystems. We will give evidence that this mechanism can be initiated without any bifurcation by the continuous movement of one of the unstable periodic orbit out of the vicinity of $x_n = y_n$ subspace.

We consider the non-identity effect on dynamics of the system (1) at $\epsilon_1 = \epsilon_2 = \epsilon$ with the detuning between parameters $\lambda_{1,2}$: $\lambda_1 = \delta \cdot \lambda, \lambda_2 = \lambda$ (where δ is the non-identity parameter). It should be noted here that the same results can be obtained when one considers $\lambda_1 = \lambda_2$ and $\epsilon_1 \neq \epsilon_2$.

In the identical case all unstable periodic orbits embedded in the chaotic attractor are transversely stable so the synchronized state is normally hyperbolic and hence it will persist under small symmetry breaking perturbations. Generally one can expect a loss of chaos synchronization due to the chaotic attractor deforming away from the synchronized state in continuous fashion. However, our investigations of bifurcations of unstable periodic trajectories have shown that the mechanism of the loss of the chaos synchronization can be different from these expectations as well as from the mechanism described for the identical case. We give evidence that in the weakly non-identical systems the initial stage of the loss of synchronization can go softly without any bifurcations of saddle periodic orbits embedded in the chaotic attractor, and it is due to the smooth shift of one saddle periodic orbits in the normal direction out of the chaotic attractor in the vicinity of $x_n = y_n$ subspace. Finally, the bifurcation of this orbit into the periodic attractor destroys the chaotic attractor in the vicinity of $x_n = y_n$ subspace.

The detuning between parameters $(\delta \neq 1)$ destroys the symmetry of the system (1). However, in slightly non-identical subsystems the oscillation regimes are modified in the weak degree. We shall determine the chaotic regime as synchronous, if $|x_n - y_n| < \Delta$, where Δ is the given small value with respect to the intensity of the chaotic oscillation. The range of acceptable values of Δ depends on the nature of the considered system and has to be estimated experimentally. In this sense the chaotic attractor A^0 corresponds to the regime of synchronization, and it is located in the vicinity of the subspace $x_n = y_n$ of the whole phase space of the system.

Let us investigate the oscillation regimes in the system (1) depending on the coupling coefficient ϵ at $\lambda = 1.56$ and consider $\delta = 0.995$ and $\Delta = 0.01$.

At this value of λ the individual system demonstrates the regime of the one-band chaotic attractor for the whole range of considered values of ϵ. Our numerical results showed that nearly synchronous motions take place in the interval of values ϵ from 0.2 to 0.55 approximately where the coupled systems demonstrate almost identical behavior. The difference between dynamical variables does not exceed the given threshold $\Delta = 0.01$. When ϵ leaves the mentioned interval, modulus of the difference $|x_n - y_n|$ increases.

The considered chaotic attractor A^0, which corresponds to the regime of the chaos synchronization, is formed as a result of the cascade of period-doubling bifurcations of orbits $2^N C^0$ located in the vicinity of $x_n = y_n$ subspace. At $\lambda = 1.56$ in the mentioned ϵ-interval [0.2,0.55] these periodic orbits are saddle ones. They are embedded in A^0 and determine the skeleton of the attractor. Fig. 1 shows saddle orbits C^0, $2C^0$ and $4C^0$ at the value $\epsilon = 0.22$ which is among the region of the chaos synchronization. As we can see they are located on the line which almost coincides with the symmetric subspace in the case of identical systems.

As in the case of identical systems [26] the behavior of the saddle point C^0 embedded in the chaotic attractor A^0 determines the initial stage of the loss of the chaos synchronization, but in both cases this behaviour can be completely different.

With the decrease of λ we observe the same mechanism of the loss of chaos synchronization as determined in [26] for identical systems (for details see [27]). A completely new mechanism is observed with the increase of the coupling coefficient. When ϵ exceeds the value 0.55 the saddle point C^0 exits from the vicinity of $x_n = y_n$ subspace but the other saddle periodic orbits stay there as their coordinates are almost not changed. As the result of it, the attractor skeleton is deformed and protrudes from the vicinity of $x_n = y_n$ subspace and chaotic attractor A^0 as seen in Fig. 2 for $\epsilon = 0.79$. Thus, with the increase of the coupling the loss of the chaos synchronization does not start with the bifurcation of the saddle point C^0, but arises from the displacement of this unstable fixed point which leaves the vicinity of

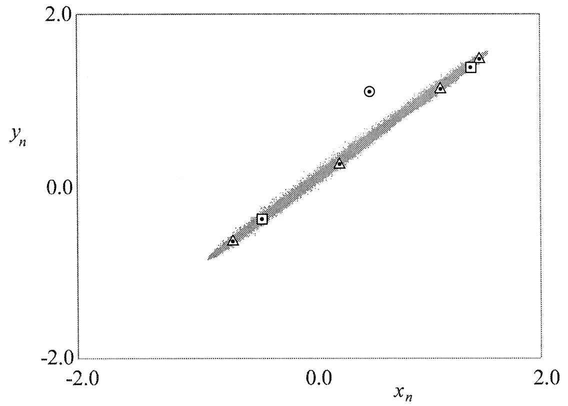

Fig. 2. The initial stage of the loss of chaos synchronization mechanism; the repeller $C^0(\circ)$ leaves the A_0 attractor located in the vicinity of $x_n = y_n$ subspace, the saddles $2C^0(\square)$; $4C^0(\triangle)$, $\epsilon = 0.790$.

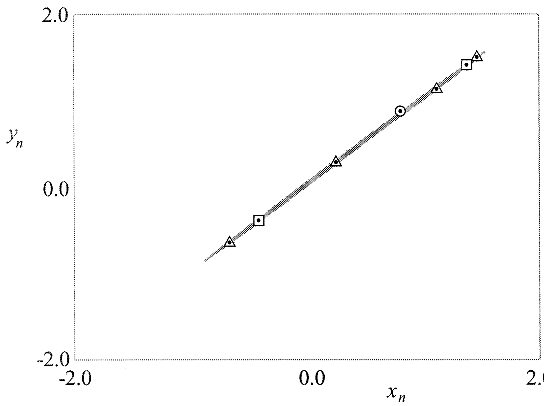

Fig. 1. Chaotic attractor A_0 and unstable periodic orbits embedded in the chaotic set A^0: the repeller $C^0(\circ)$, the saddles $2C^0(\square)$; $4C^0(\triangle)$; $\lambda = 1.56$, $\epsilon = 0.22$.

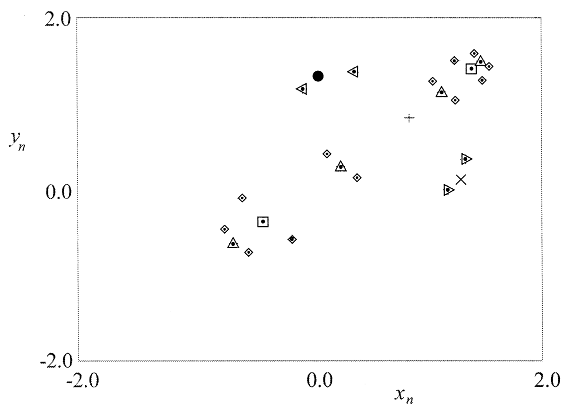

Fig. 3. Periodic attractors: C^0;($\bullet$), C_2^0;($\times$) and unstable periodic orbits: the repellers $C_1^0(+)$, $2C^0(\square)$; $4C^0(\triangle)$ and saddle orbits with doubled period located in the neighbourhoods of periodic attractors and repellers after destruction of the chaotic attractor A_0; $\lambda = 1.56$, $\epsilon = 0.857$.

$x_n = y_n$ subspace. For $\epsilon \in [0.8069, 0.8448]$ one observes various bifurcations of unstable periodic orbits embedded in the chaotic attractor A_0 [27]. The saddle periodic orbits $2C^0, 4C^0, 8C^0, 16C^0$ undergo the period-doubling bifurcation, respectively. They are transformed to repellers, and the doubled period orbits appear in their vicinity. The period 0ne repeller C_1^0 and saddle C_1^0 appear in the vicinity of $x_n = y_n$ subspace. These bifurcations lead to the more developed bubbling attractor. At $\epsilon = 0.8448$ the maximum eigenvalue of the saddle fixed point C^0 becomes equal to -1. As a result C^0 transforms to a stable point (Fig. 3), and in its neighborhood the saddle orbit of doubled period softly appears. This bifurcation corresponds to the reversed subcritical period-doubling bifurcation. At $\epsilon = 0.8494$ the fixed point C_s^0 undergoes the similar bifurcation. As a result it becomes the stable fixed point and in its vicinity the saddle periodic orbit softly appears. The bubbling attractor becomes the non-attracting chaotic set (chaotic attractor A_0 in the vicinity of $x_n = y_n$ subspace disappears) and the process of the loss of the chaos synchronization is completed.

In this work we studied dynamics of two coupled non-identical logistic maps. Particularly, the process of the loss of the chaos synchronization accompanied by bubbling transition has been investigated. It was shown that in the weakly non-identical systems the process of losing stability of the synchronous regime can be different from that identified for the identical system, i.e. can be initiated without any bifurcations of unstable periodic trajectories embedded in the chaotic attractor, and it is due to the smooth shift of one saddle periodic orbit from the chaotic attractor located at the vicinity of $x_n = y_n$ subspace. Period-doubling bifurcations and saddle-node ones (the birth of new periodic orbits) complete the process of the loss of the chaos synchronization. We hope that this mechanism is typical for the wide class of coupled weakly non-identical systems, it can be observed experimentally, and it has to be taken into account in designing experimental systems based on chaos synchronization.

Acknowledgements

This work has been supported by the KBN (Poland) under the project no. 7TO7A 039 10.

References

[1] T. Yamada, H. Fujisaka, Progr. Theor. Phys. 70 (1983) 1240.
[2] A.S. Pikovsky, Z. Phys. B 55 (1984) 149.
[3] S.P. Kuznetsov, Izv. Vuzov. Radiofizika 28 (1985) 991.
[4] V.S. Afraimovich, N.N. Verichev, M.I. Rabinovich, Radiophysics and Quantum Electron. 29 (1986) 795.
[5] V.V. Astakhov, B.P. Bezruchko, V.I. Ponomarenko, E.P. Seleznev, Izv. Vuzov. Radiofizika 31 (1988) 627.
[6] L.M. Pecora, T.L. Carrol, Phys. Rev. Lett. 64 (1990) 821.
[7] T. Endo, L.O. Chua, Int. J. Bif. Chaos 1 (1991) 701.
[8] M. de Sousa Viera, A.J. Lichtenberg, M.A. Lieberman, Phys. Rev. A 46 (1992) 7359.
[9] T. Kapitaniak, Phys. Rev. E 50 (1994) 1642.
[10] V.S. Anishchenko, T.E. Vadivasova, D.E. Postnov, M.A. Safonova, Radioeng. and Electron. 36 (1991) 338.
[11] P.S. Landa, M.G. Rosenblum, Sov. Phys. Dokl. 37 (1992) 237.
[12] L. Kocarev, A. Shang, L.O. Chua, Int. J. Bif. Chaos 3 (1993) 479.
[13] M. Rosenblum, A. Pikovsky, J. Kurths, Phys. Rev. Lett. 76 (1996) 1804.
[14] N.F. Rulkov, K.M. Sushchik, L.S. Tsimring, H.D.I. Abarbanel, Phys. Rev. E 51 (1995) 980.
[15] H. Fujisaka, T. Yamada, Progr. Theor. Phys. 74 (1985) 917.
[16] A.S. Pikovsky, P. Grassberger, J. Phys. A 24 (1991) 4587.
[17] P. Ashwin, J. Buescu, I. Stewart, Phys. Lett. A 193 (1994) 126.
[18] S.C. Venkataramani, B.R. Hunt, E. Ott, Phys. Rev. E 54 (1996) 1346.
[19] Y. Maistrenko, T. Kapitaniak, Phys. Rev. E 54 (1996) 1.
[20] E. Ott, J.C. Sommerer, Phys. Lett. A 193 (1994) 126.
[21] E. Ott, J.C. Sommerer, J.C. Alexander, I. Kan, J.A. Yorke, Physica (Amsterdam) 76 D, 384 (1994).
[22] N. Platt, E.A. Spigel, C. Tresser, Phys. Rev. Lett. 70 (1993) 279.
[23] Y.-C. Lai, C. Grebogi, J.A. Yorke, S.C. Venkataramani, Phys. Rev. Lett. 77 (1996) 55.
[24] N.F. Rulkov, M.M. Suschik, Phys. Lett. A 214 (1996) 145.
[25] A. Pikovsky, G. Osipov, M. Rosenblum, M. Zaks, J. Kurths, Phys. Rev. Lett. 79 (1997) 47.
[26] V. Astakhov, A. Shabunin, T. Kapitaniak, V. Anishchenko, Phys. Rev. Lett. 79 (1997) 1014.
[27] V. Astakhov, M. Hasler, T. Kapitaniak, A. Shabunin, V. Anishchenko, Phys. Rev. E58 (1998) 5620.