PERGAMON

Chaos, Solitons and Fractals 11 (2000) 2505–2510

CHAOS
SOLITONS & FRACTALS

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Impact force generator: self-synchronization and regularity of motion

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Abstract

Impacts in multibody mechanical systems are an object of interest for many scientists in the world. In this paper, we present a principle of operation of the impact force generator being an element of the rotor of the heat exchanger. In this machine, step disturbances of the rotational velocity of the generator cause rapid changes of the rotational velocity of the exchanger rotor, which leads to the intensification of the heat exchange process. We show the phenomenon of self-synchronization, regular motion of the system, and in a special case: chaotic motion of the rotor. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

In many industrial machines, the impact of their movable parts is either the basic principle of their operation or the effect which improves their operating efficiency. The classic examples of such machines or devices are: a pneumatic hammer, impact dampers, or heat exchangers. Lately the behavior of the bridges during an earthquake is also investigated using the mathematical model with impacts. In the heat exchanger, one of the factors that contribute to intensification of the exchange process are disturbances in the rotational velocity of its rotor. These disturbances may have a character of step disturbances. The simplest way to generate such disturbances is to employ the phenomenon of impact which causes, according to the Newton's hypothesis, step variations of the velocity of the bodies impacting on each other. These disturbances can have (depending on the assumed parameters of the generator) a periodical or chaotic character [2,3].

The impact force generator consisting of an engine connected with the rotor by means of an elastic shaft, was the object of investigations, results of which were presented in [1–3]. Below, a principle of operation of another type of generator of step disturbances has been presented. The system now consists of the engine, which is connected firmly with the rotor of the generator, and of the exchanger rotor, which is connected with the generator by means of the elastic shaft. Two selected aspects of the generator operation have been shown: the self-synchronization phenomenon and the conditions, whose fulfillment leads to the maximization of the amplitude of the rotor acceleration.

2. Physical model of the generator

The object of considerations is a system composed of three parts (Fig. 1):
- a rotor equipped with a fender, driven by an electric engine,

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PII: S0960-0779(99)00157-5

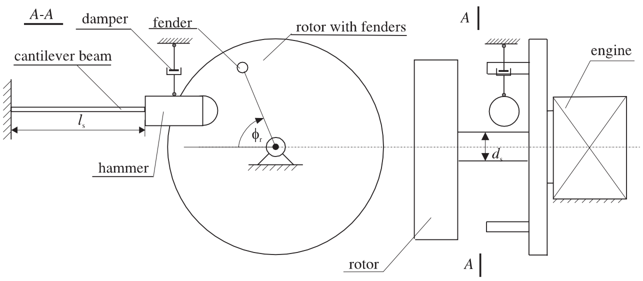


Fig. 1.

- a hammer in the form of a cylinder with a semicircular end, mounted on the end of a cantilever beam,
- a rotor of the exchanger connected with generator by means of the elastic shaft.

During the operation of the generator, the rotor fender impacts on the hammer, which causes vibrations of the hammer on one hand and the desired step variations of the rotational velocity of the rotor of the generator on the other. These variations cause the sudden (but not step) changes of the rotational velocity of the exchanger. The hammer vibrations are suppressed by a viscous damper. The masses of the cantilever beam, the damper and the elastic shaft have been neglected. The drive engine is connected firmly with the rotor of the generator. When the disturbances of the rotational velocity of the engine are small (less than a few per cent) and the average value of this velocity is not far from the synchronical one, the driving moment may be taken from the static characteristic of the engine.

Because of a rather sophisticated geometry of the system hammer – rotor with fenders, two kinds of impacts occur during its operation:

1. the fender collides with the cylindrical part of the hammer, and the line of impact is perpendicular to the hammer axis,
2. the fender collides with the spherical part of the hammer, and the line of impact goes through the center of the fender and the center of the basis of the spherical part of the hammer.

The mathematical model of the system consists of rather obvious equations of impacts, which are based on Newton's law, and were presented in [1], and of the equations of motion. Equations of motion of the hammer, written in the Cartesian system of coordinates which is connected with the center of gravity of the hammer in the static equilibrium position, are as follows:

$$\begin{bmatrix} m & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\phi} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} y - a\phi \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{1}$$

where

$$[K_{ij}] = \begin{bmatrix} \dfrac{12EI}{l_s^3} & \dfrac{-6EI}{l_s^2} \\ \dfrac{-6EI}{l_s^2} & \dfrac{4EI}{l_s} \end{bmatrix}. \tag{2}$$

In the above equations, the fact is taken into account that the point at which the spring is joined to the mass of the hammer is at the distance a from the gravity center. As it is very difficult to define the real character and intensity of damping of the vibrations, the matrix of damping has a simplified form: the damping coefficients C_{11} are calculated from the assumed logarithmic decrement of damping Δ.

Equations of motion of the rotor system, written in the coordinate system connected with the axis of the rotations, are as follows:

$$\begin{bmatrix} B_r & 0 \\ 0 & B_e \end{bmatrix} \begin{Bmatrix} \ddot{\phi}_r \\ \ddot{\phi}_e \end{Bmatrix} + \begin{bmatrix} C_\phi & -C_\phi \\ -C_\phi & C_\phi \end{bmatrix} \begin{Bmatrix} \dot{\phi}_r \\ \dot{\phi}_e \end{Bmatrix} + \begin{bmatrix} K_\phi & -K_\phi \\ -K_\phi & K_\phi \end{bmatrix} \begin{Bmatrix} \phi_r \\ \phi_e \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_e(\dot{\phi}_e) \end{Bmatrix}. \tag{3}$$

The coefficient C_ϕ is calculated from the assumed logarithmic decrement of damping of the torsional vibrations Δ_ϕ. M_e is the driven moment of the engine.

3. Data of the system

The basic data of the considered system are as follows:

- *The hammer*: Inertial moment $B = 12.14 \times 10^{-6}$ kg m^2, mass $m = 89.83 \times 10^{-3}$ kg, radius $R = 0.01$ m, length $l_h = 0.03$ m, cross-section of the beam $I = 63.6 \times 10^{-12}$ m^4.
- *The rotor system*: Inertial moment of the exchanger rotor $B_r = 0.015$ kg m^2, inertial moment of the engine and rotor with fender $B_e = 0.015$ kg m^2, distance fender – axis of rotations $r = 0.1$ m, length of the elastic spring $l = 0.2$ m.

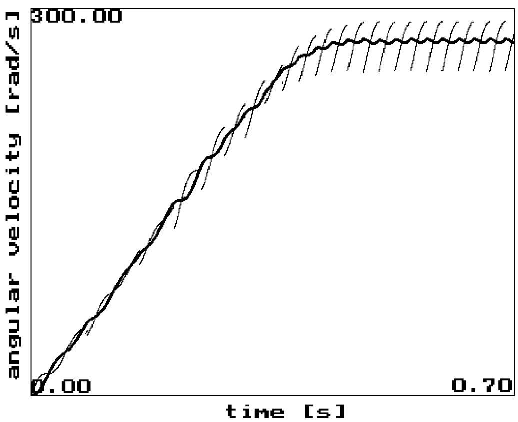
4. Principle of operation

A time diagram representing the motion of the rotor system is shown in Fig. 2. The time of the motion is represented on the horizontal axis, and the rotational velocity of the rotor with fenders (thin line) and of the exchanger rotor (thick line) on the vertical axis. As can be seen, during each rotation of the rotor, its fender collides with the hammer causing the hammer vibrations and step variations of the angular velocity of this rotor. Due to the elasticity of the shaft, the variations of the rotational velocity of the exchanger are of the smaller amplitude, and do not have a step character.

5. Influence of the length of the cantilever beam on the system operation

Fig. 3 presents a map of impacts for many systems in which generators differ in the length of the cantilever beam l_s ($\Delta = \ln(3)$, $\Delta_\phi = \ln(2)$ and $d_s = 0.2$ m). The values of the angular velocity of the rotor before (ω_{bi}) and after (ω_{ai}) impacts and, additionally, the curves showing the values of the fractions of the basic frequency of free vibrations of the hammer: $1/2\alpha_1$ and $1/3\alpha_1$ have been shown on this map. In Fig. 3, a close relation between the variations of both the velocities ω_{ai} and ω_{bi} and the function of α_1 can be easily observed. For instance: when $l_s = 0.0725$ m, the rotor velocity ω_{bi} (close to its average angular velocity) is equal to approximately 1/3 of the value of the basic frequency of free vibrations of the hammer α_1. It means that the impact forcing of these vibrations has a subharmonic frequency. Under such conditions, the generation of the hammer vibrations occurs at minimal values of the impact impulse. As a consequence, the values of ω_{bi} and ω_{ai} are high, while the difference between them – minimal.

An increase in the cantilever beam length (up to 0.085 m) causes of course a decrease in the value of $1/3\alpha_1$. Despite it, however, the hammer 'wants' the impact forcing of its free vibrations to have a



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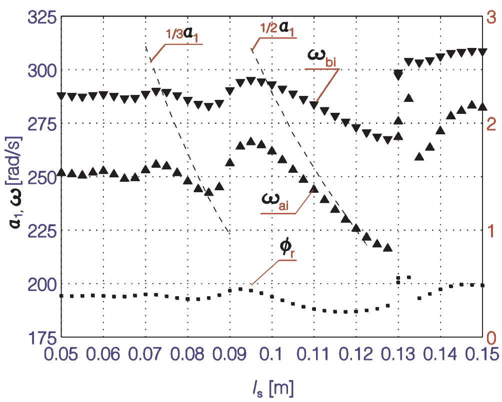

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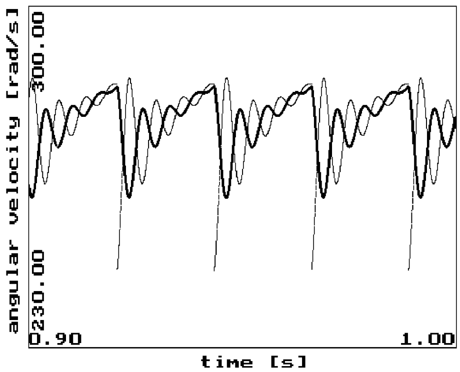
Fig. 2.

Fig. 3.

subharmonic character: the impacts are stronger and stronger, which, as a consequence, causes the diminishing of both ω_{bi} and ω_{ai}. This way of affecting the angular velocity by the hammer has been called the self-synchronization of the system. When l_s exceeds the value 0.1 m, the conditions for easy generation of the hammer vibrations with the next subharmonic frequency $1/2\alpha_1$, arise and the situation repeats, up to $l_s = 0.13$ m. As can be seen, a choice of the length l_s makes it possible to control both the average velocity of the rotor and the intensity of the impacts.

6. Influence of the elastic shaft on the rotor acceleration

Fig. 4 shows a part of the time diagram of the system (analogous to the one in Fig. 2) for $l_s = 0.11$ m and $d_s = 0.02$ m. We observe the discontinuities in the thin line (step changes of the velocity), and changes of the velocities due to free torsional oscillations of the rotor system. For the exchange process a maximum value of the acceleration of the exchanger rotor plays a great role. This acceleration is shown in Fig. 5 as the function of the rotational velocity. Comparing this figure with Fig. 6 ($d_s = 0.01$ m) and Fig. 7 ($d_s = 0.04$ m) enables us to establish the influence of the stiffness of the elastic shaft k_s on the exchanger acceleration.



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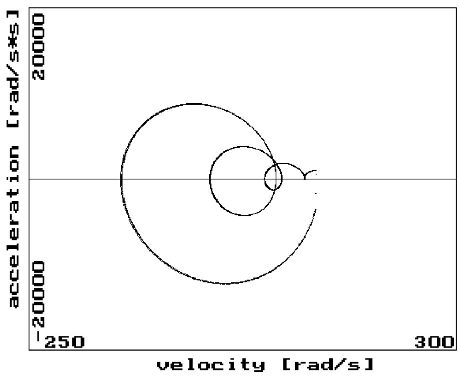

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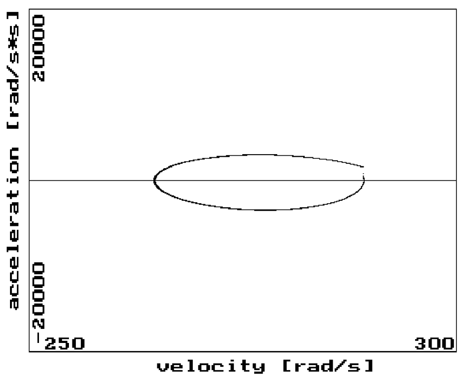
Fig. 4.
Fig. 5.



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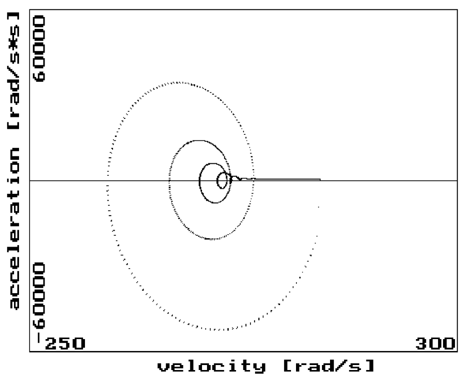

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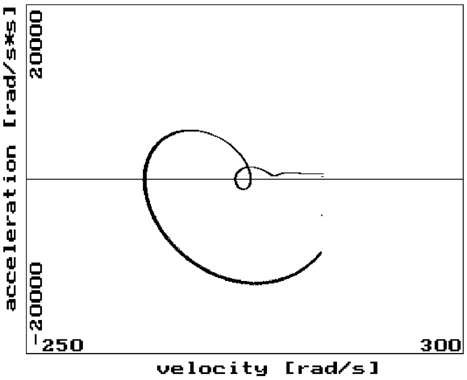
Fig. 6.
Fig. 7.



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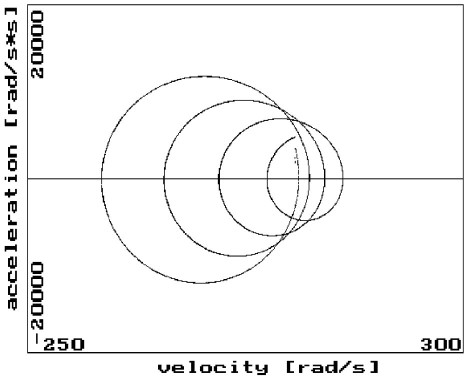

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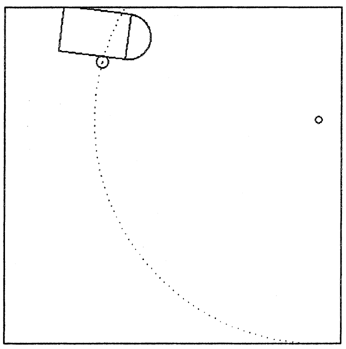
Fig. 8.
Fig. 9.



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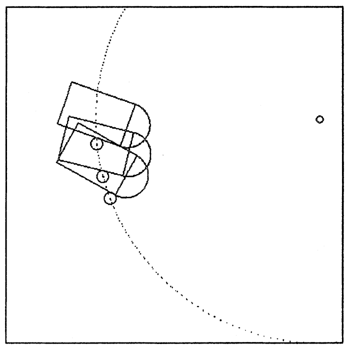

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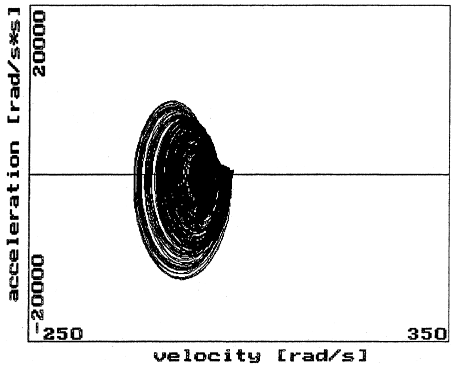
Fig. 10.
Fig. 11.

The results presented above were obtained for the logarithmic decrement $\Delta_\phi = \ln(2)$. The next two figures present influence of this decrement on the acceleration. As it can be seen, the amplitude of acceleration hardly depends on the Δ_ϕ. For greater damping (Fig. 8, $\Delta_\phi = \ln(4)$) free vibrations diminish very quickly, while for the small damping (Fig. 9, $\Delta_\phi = \ln(1.2)$) their amplitude diminishes slowly. In the last case we have one impact during one rotation of the rotor, but up to four distinguished changes of the acceleration, which is better for the exchange process.

7. Regularity of motion

Fig. 10 shows the configuration of the hammer and fender during the impact for the case $l_s = 0.11$ m and $d_s = 0.02$ m. As we see, all the impacts are identical, so we observe the regular motion with period 1. Such state guarantees the stable and safe operation of the system. After introducing longer cantilever beam or diminishing the damping we may observe the period multiplying (Fig. 11 – period 3 for $l_s = 0.1825$ m) or the motion of the system may even become irregular (Figs. 12 and 13 – chaos for $l_s = 0.1875$ m). Such operation conditions are very dangerous: during the chaotic motion of the system the impact of the second kind occurs, which may lead to the damage of the cantilever beam.



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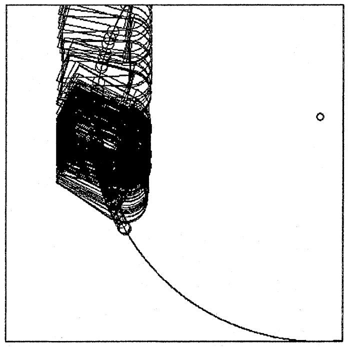

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Fig. 12.

Fig. 13.

8. Conclusions

The object of the numerical investigations presented here is a mechanical impact force generator. During the investigations it was found that in the majority of cases the system exhibited a regular motion and it is possible to control both the intensity of impacts and the average value of the rotational velocity. By a proper choice of the value of the stiffness coefficient of the elastic shaft connecting the generator and the exchanger we may control the amplitude of the exchanger acceleration. When the frequency of impacts is close to the first natural frequency of hammer vibrations α_1, it is possible to have the system with the features of a chaotic motion.

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