PERGAMON

Chaos, Solitons and Fractals 12 (2001) 1823–1826

CHAOS
SOLITONS & FRACTALS

www.elsevier.nl/locate/chaos

Antiphase synchronization of chaos by noncontinuous coupling: two impacting oscillators

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Accepted 22 June 2000

Abstract

We show that two identical chaotic oscillators can evolve in antiphase synchronization regime when noncontinuous coupling between them is introduced. As an example, we consider dynamics of two mechanical oscillators coupled by impacts. © 2001 Elsevier Science Ltd. All rights reserved.

Coupled nonlinear oscillators are of great interest not only in engineering but also in biology, chemistry and physics as well. [1] Over the last decade, the field of synchronization of coupled chaotic systems has advanced considerably due to the development of analytical methods and the rapid improvement in experimental techniques [2–4]. Since then, synchronization has become one of the most active fields of research in nonlinear dynamics.

In the paper [5], Cao and Lai showed that the new class of synchronism exists in chaotic systems with symmetry. They pointed out that there exists such a decomposition of the state variables which allows the slaving subsystem to synchronize with its replica in amplitude but with opposite sign for initial conditions chosen from large regions in the phase space.

Consider two dynamical systems $\dot{x} = f(x)$ and $\dot{y} = f(y)$ where $x, y \in \mathbb{R}^n$. The x-system is synchronized with the y-system if $\lim_{t\to\infty} |x - y| \to 0$. x and y systems are synchronized with antiphase when $x = -y$ for $t \to \infty$. In a great number of papers it was shown that two chaotic systems can be synchronized when continuous linear or nonlinear coupling between them is introduced.

In this brief report, we show that two identical antiphased forced chaotic systems can evolve in antiphase synchronization regime when noncontinuous coupling between them is introduced. As an example, we consider two mechanical oscillators coupled by impacts. Impact oscillators have been recently widely studied [6–10] as they have some potential applications in mechanical and civil engineering. Such systems exhibit Feigenbaum scenario [6–8] and intermittent routes to chaos sudden changes in the chaotic attractor, Devil's attractors [8] as well as the different types of grazing bifurcations [9,10].

Consider the system shown in Fig. 1. Two masses m are suspended on two identical springs with stiffness coefficient k and two dampers with damping coefficient c. Periodic forces $F \sin \omega_1 t$ and $F \sin \omega_2 t$ acting on both masses caused the chaotic oscillations of both oscillators. Consider that the forcing is in a counter phase, i.e., $\omega_1 t = \omega_2 t + \pi$ and $\omega_1 = \omega_2$. Assume that the distance between masses 1 and 2, l can slowly decrease allowing impacts between masses 1 and 2.

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[1] There is a large literature on coupled oscillators. For a list of representative papers in various fields, see [1].

PII: S0960-0779(00)00145-4

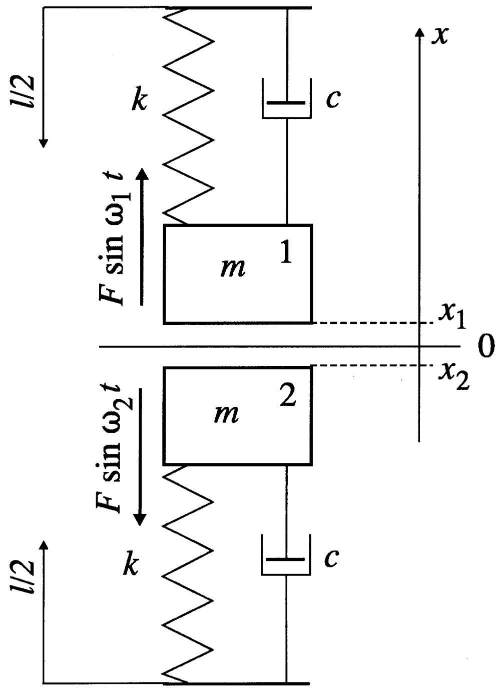

Fig. 1. Model of two mechanical oscillators.

This system can be described by the following dimensionless equations:

$$\begin{aligned} &\ddot{x}_1 + a\dot{x} + x_1^3 = b\,\sin(\eta\tau), \\ &\ddot{x}_2 + a\dot{x}_2 + x_2^3 = b\,\sin(\eta\tau + \pi), \end{aligned} \tag{1}$$

where $a = c(km)^{-1/2}\bar{x}^{-1}, b = F/k\bar{x}^{-3}, \eta = \omega/\alpha, \alpha = (km)^{1/2}\bar{x}$ and $\bar{x}$ is a scale of $x_{1,2}$. Equations describing the impacts based on the well-known Newton's law base are as follows:

$$\begin{aligned} &\dot{x}_1' - \dot{x}_1 = S, \\ &\dot{x}_2' - \dot{x}_2 = -S, \\ &\dot{x}_1' - \dot{x}_2' = -k_r(\dot{x}_1 - \dot{x}_2), \end{aligned} \tag{2}$$

where $S = S_d/m\alpha$ denotes the impulse of the force (S_d is dimensional impulse), k_r is a restitution coefficient and $()'$ describe the velocities before impacts.

In our numerical calculations, we consider $a = 0.1, b = 10, \eta = 1$ and $k_r = 0.9$. We assumed that both the oscillators evolve independently (without impacts) on their chaotic attractors and allowed the distance between them to δ to decrease slowly. Slowly decreasing $\delta = l/\bar{x}$ one observes that the impacts between masses 1 and 2 appear for $\delta < \delta_i = 7.242$. Initially the oscillations of both masses are uncorrelated. Bifurcation diagram shown in Fig. 2(a) indicates that for different values of δ both chaotic and periodic behavior is possible. With the decrease of δ, for $\delta < \delta_s = 6.945$ we observe antiphase synchronization regime $x_1 = -x_2$ which is robust in the large interval of δ as can be seen in $|x_1 + x_2|$ versus δ plot shown in Fig. 2(b). Such oscillations are shown in Figs. 3(a) and (b), where time series (Fig. 3(a)) and x_1 versus x_2 plot (Fig. 3(b)) clearly indicates antiphase synchronized oscillations. Antiphase synchronization of two oscillators occurs on the chaotic attractor which is different from the one of the impactless system, but on the one which is located in the same region of the phase space, and it is preserved inside periodic windows.

To summarize, we have shown that the impacts between oscillators can correlate their oscillations. This phenomenon is robust and can be observed in a large class of mechanical impacting systems. It can also be

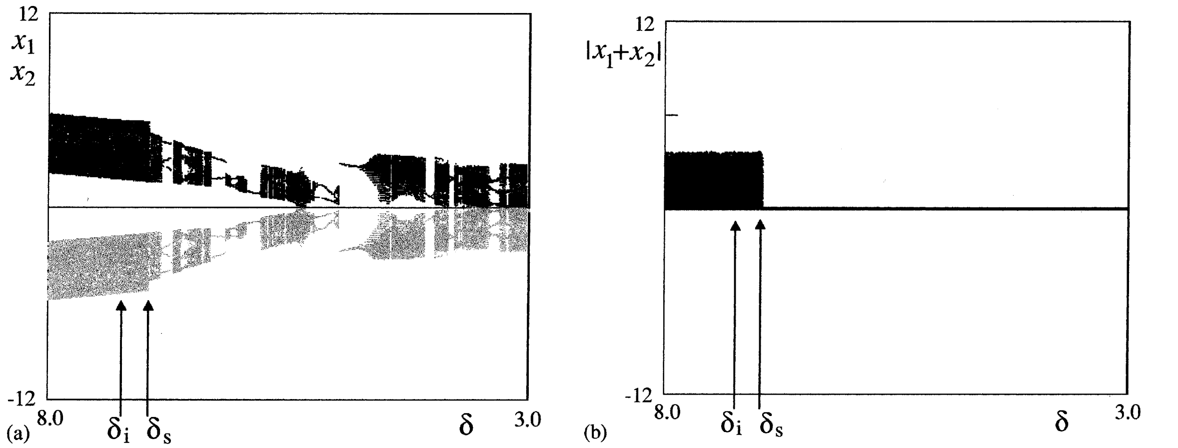

Fig. 2. Bifurcation diagram of the system (1) with the dimensionless distance between oscillators δ, as a control parameter; $a = 0.1, b = 10, \eta = 1$ and $k_r = 0.9$; (a) x_1 (black) and x_2 (grey) versus δ; (b) $|x_1 + x_2|$ versus δ.

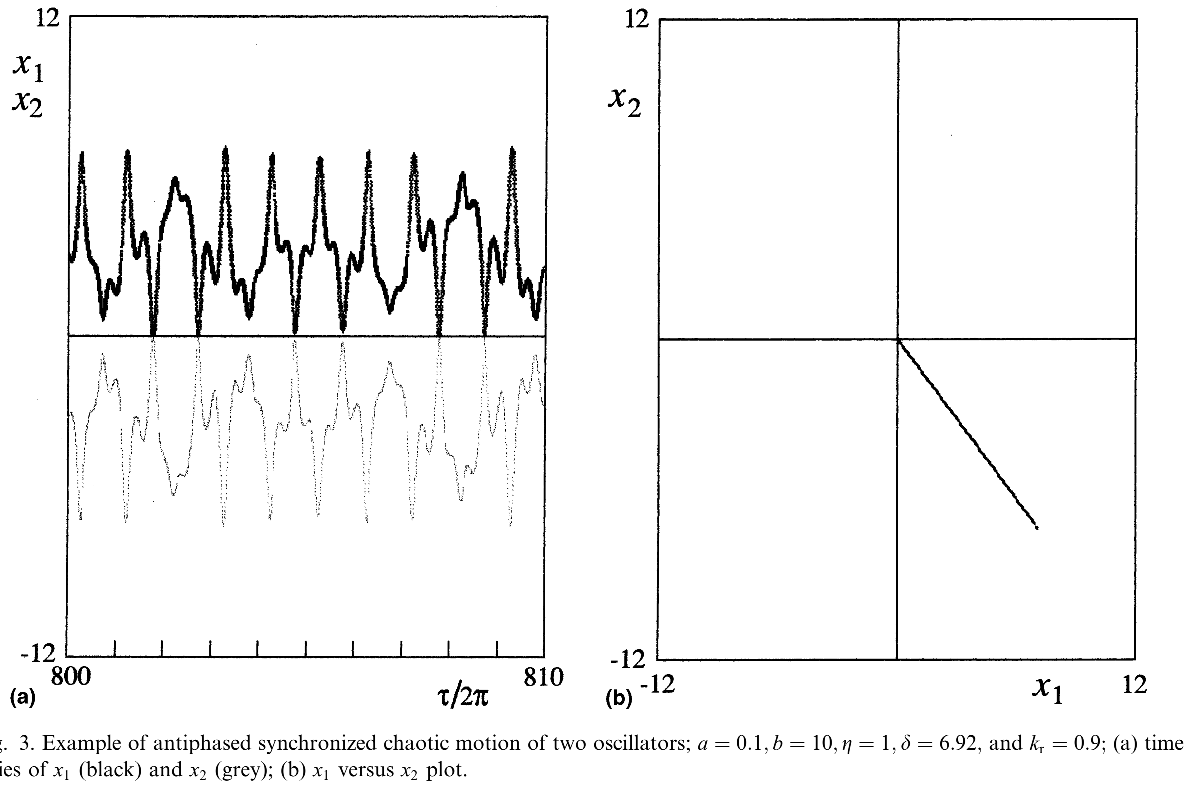

Fig. 3. Example of antiphased synchronized chaotic motion of two oscillators; $a = 0.1, b = 10, \eta = 1, \delta = 6.92$, and $k_r = 0.9$; (a) time series of x_1 (black) and x_2 (grey); (b) x_1 versus x_2 plot.

observed in other nonsmooth systems, where the introduction of noncontinuous coupling can lead to the antiphase synchronization of subsystems.

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