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Letter to the Editor

Synchronization of mechanical systems driven by chaotic or random excitation

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1. Introduction

The phenomenon of synchronization in dynamical and, in particular, mechanical systems has been known for a long time. Recently, the idea of synchronization has been also adopted for chaotic systems. It has been demonstrated that two or more chaotic systems can synchronize by linking them with mutual coupling or with a common signal or signals [1–5,15]. In the case of linking a set of identical chaotic systems (the same set of ODEs and values of the system parameters) ideal synchronization can be obtained. The ideal synchronization takes place when all trajectories converge to the same value and remain in step with each other during further evolution (i.e., $\lim_{t\to\infty} |x(t) - y(t)| = 0$ for two arbitrarily chosen trajectories $x(t)$ and $y(t)$). In such a situation all subsystems of the augmented system evolve on the same attractor on which one of these subsystems evolves (the phase space is reduced to the synchronization manifold). Linking homochaotic systems (i.e., systems given by the same set of ODEs but with different values of the system parameters) can lead to practical synchronization (i.e., $\lim_{t\to\infty} |x(t) - y(t)| \leqslant \varepsilon$, where ε is a vector of small parameters) [6,7]. In such linked systems it can also be observed that there is a significant change of the chaotic behaviour of one or more systems. This so-called "controlling chaos by chaos" procedure has some potential importance for mechanical and electrical systems. An attractor of such two systems coupled by a negative feedback mechanism can be even reduced to the fixed point [8].

This paper concerns the ideal synchronization of a set of identical uncoupled mechanical oscillators (with one or more degrees of freedom) linked by common external excitation only. This problem has been described widely for oscillators with periodic excitation because such kind of driving is often met in real oscillators [1]. However, from a viewpoint of practical considerations, a non-periodic external excitation can also occur in mechanical systems. For that reason, this paper concentrates on the analysis of the ideal synchronization for non-linear oscillators forced by chaotic and stochastic external driving. The analysis presented is based on the connections

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between the appearance of synchronization and Lyapunov exponents [9,12] and is motivated by Pecora and Carroll's theoretical and experimental studies [3,4]. They have discovered that the chaotic systems linked by common signals can synchronize if the Lyapunov exponents for the subsystems are all negative. The present considerations, supported by the numerical analysis, show that even a non-periodic nature of common external excitation can lead to synchronization of driven mechanical oscillators. The necessary, but not sufficient, condition for occurrence of synchronization is the negative sign of the Lyapunov exponents associated with the response of the system. In the numerical experiment reported here, a pair of non-linear mechanical oscillators of the Duffing type, forced by a irregular deterministic signal or random excitation has been used.

2. Synchronization and Lyapunov exponents

Consider a set of the k-number of separate identical m-degree-of-freedom mechanical oscillators forced by common external excitation, as shown schematically in Fig. 1. The assumed character of driving vibrations (function $e(t)$) is chaotic and the differential equations describing the motion of oscillators can be chosen arbitrarily (linear or non-linear, smooth or non-smooth). The dynamical state of each of these oscillators is determined by the n-dimensional vector $\mathbf{z}_i = [z_{i1}, z_{i2}, \dots, z_{in}]$ $(i = 1, 2, \dots, k)$. This vector describes a response of the system. The s-dimensional vector $\mathbf{e} = [e_1, e_2, \dots, e_s]$ describes an evolution of the excitation. Thus, the state of a separate subsystem is described in the phase space of $r = (n + s)$ dimensions and the equations of motion of such a subsystem can be written in the general first order differential equation autonomous form

$$\dot{\mathbf{z}}_i = \mathbf{f}(\mathbf{z}_i, \mathbf{e}), \tag{1a}$$

$$\dot{\mathbf{e}} = \mathbf{f}(\mathbf{e}) \tag{1b}$$

or the non-autonomous form

$$\dot{\mathbf{z}}_i = \mathbf{f}(\mathbf{z}_i, \mathbf{e}, t), \tag{2}$$

where $\mathbf{z}_i \in \mathbf{R}^n$ $(i = 1, 2, \dots, k)$ and $\mathbf{e} \in \mathbf{R}^s$

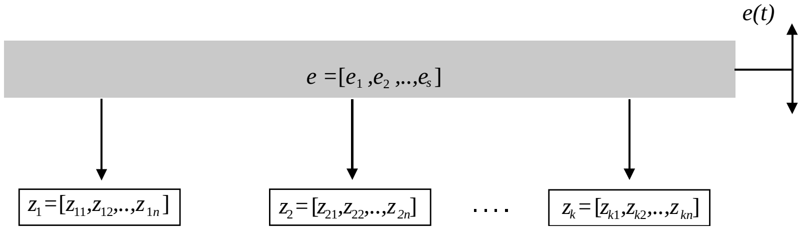

Fig. 1. The scheme of the system under consideration.

Hence, for the entire system of oscillators, the equations of motion are as follows:

$$\begin{aligned}\dot{\mathbf{z}}_1 &= \mathbf{f}(\mathbf{z}_1, \mathbf{e}, t),\\ \dot{\mathbf{z}}_2 &= \mathbf{f}(\mathbf{z}_2, \mathbf{e}, t),\\ \ldots &= \ldots\ldots\ldots,\\ \dot{\mathbf{z}}_k &= \mathbf{f}(\mathbf{z}_k, \mathbf{e}, t).\end{aligned} \tag{3}$$

Eq. (1b) describes the dynamical evolution of excitation in the *s*-dimensional subspace of the system phase space (*excitation subspace*). From the form of Eqs. (1) it results that the time evolution of excitation is independent of the remaining state variables and is characterized by the *s*-number of Lyapunov exponents, where at least one of them is equal to zero. Eq. (1a) describes the evolution of the system response (*response subspace*) in the *n*-dimensional subspace of the phase space, which is transversal to the above-mentioned excitation subspace and is characterized by a series of *n* Lyapunov exponents. A two-dimensional visualization of the system phase space is presented in Fig. 2. The entire spectrum of Lyapunov exponents of the system under consideration (Eq. (1a)) contains the exponents associated with excitation (*excitation Lyapunov exponent* —ELE or λ_e in further considerations) and response (*response Lyapunov exponent*—RLE or λ_r in further considerations), so the number of Lyapunov exponents is equal to *r*. Since excitation does not depend on the response of the system, the values of ELEs are constant and independent of the parameters of oscillators.

The non-autonomous system given by Eqs. (3) can be considered as a set of separate identical subsystems with common chaotic driving. Hence, it can be assumed that the solutions of Eqs. (3), starting from different initial conditions, represent independent trajectories $\mathbf{z}_i(t)$ (given by Eqs. (1) or (2)) evolving on the same attractor (after a period of the transient motion). Common excitation causes that a distance between these trajectories in the direction associated with zero Lyapunov

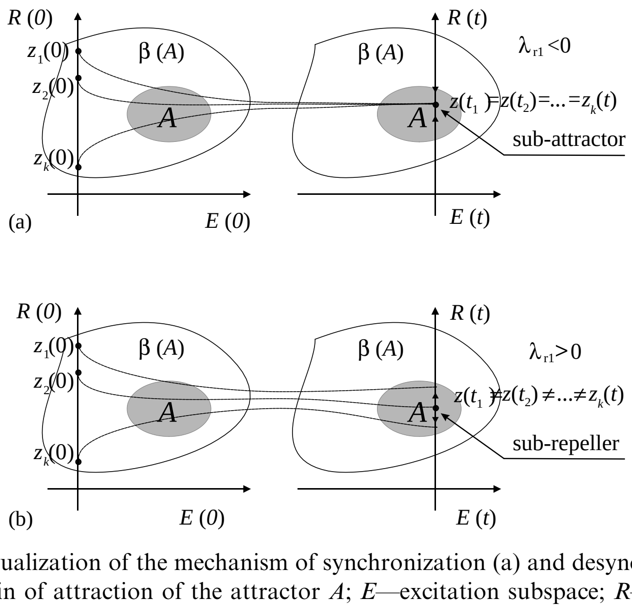

Fig. 2. Two-dimensional visualization of the mechanism of synchronization (a) and desynchronization (b); *A*—chaotic system attractor; $\beta(A)$—basin of attraction of the attractor *A*; *E*—excitation subspace; *R*—response subspace.

exponent amounts to zero and they can be found in the same n-dimensional response subspace at each moment. This fact leads to the conclusion that for a set of negative RLEs there exists a point in the response subspace which is a stable *sub-attractor*. This point is a trace of the system attractor in the response subspace. It causes that trajectories starting from different points of the basin of attraction evolve to the same state and oscillators will synchronize (Fig. 2a). In other words, an invariant subspace representing the synchronized state ($\mathbf{z}_1 = \mathbf{z}_2 = \cdots = \mathbf{z}_k$) is a stable attractor. Such synchronization is caused by common excitation only and it occurs without any additional coupling between oscillators.

If at least one RLEs is positive, then the synchronization between the oscillators under consideration is impossible because instability associated with positive RLE causes divergence of nearby trajectories (Fig. 2b) in the response subspace and the *sub-attractor* becomes a *sub-repeller* representing an unstable orbit in this subspace.

3. Numerical examples

In this section numerical investigations of the synchronization phenomenon between two identical non-linear oscillators of the Duffing type, driven by the same external forces are presented (see Fig. 3). This is the simplest example of the system of oscillators given by Eqs. (3). The presented numerical analysis is a confirmation of the theoretical considerations described in the previous section. According to Eqs. (3), the system of oscillators shown in Fig. 3 can be described by the following first order differential equations in non-autonomous form:

$$\dot{x}_1 = x_2, \tag{4a}$$

$$\dot{x}_2 = -ax_1^3 - hx_2 + qe(t), \tag{4b}$$

$$\dot{y}_1 = y_2, \tag{4c}$$

$$\dot{y}_2 = -ay_1^3 - hy_2 + qe(t), \tag{4d}$$

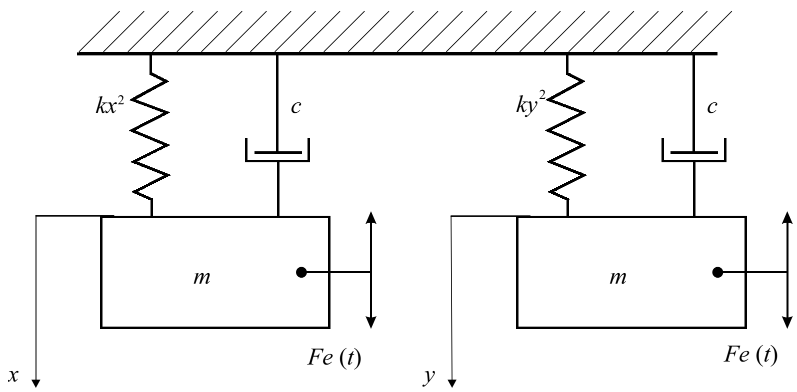

Fig. 3. Non-linear oscillators applied in numerical experiment.

where $a = k/m$, $h = c/m$, $q = F/m$ (c, k, m are the parameters of the oscillators, F is the amplitude of the driving force, for details see Fig. 3). Common dynamic excitation is realized by two identical forces acting on both oscillators and working in the same phase. In the numerical experiment reported below, the following types of external excitation have been applied: (i) excitation given by the chaotic Lorenz system, (ii) irregular discontinuous external signal, and (iii) randomly generated driving.

3.1. Chaotic excitation

In the first example a source of external excitation is the well-known Lorenz system working in the chaotic range, given by following equations:

$$\begin{aligned} \dot{e}_1 &= -\delta(e_1 - e_2), \\ \dot{e}_2 &= -e_1 e_3 + r e_1 - e_2, \\ \dot{e}_3 &= e_1 e_2 - b e_3. \end{aligned} \tag{5}$$

In this case the system under consideration (Eqs. (4a)–(4d)) is excited using a signal given by a variable e_1 from the Lorenz system, i.e., $e(t) = e_1$. The parameters of excitation are characteristic of the classical Lorenz equation: $\delta = 10$, $r = 28$, $b = 8/3$, thus the evolution of excitation in the three-dimensional excitation subspace ($s = 3$) is characterized by the spectrum of three constants ELEs ($\lambda_{e1} = 0.899$, $\lambda_{e2} = 0.000$, $\lambda_{e3} = -14.567$).

The bifurcational analysis of this system is presented in Fig. 4. In the numerical simulations, coefficient h representing damping rate has been used as a bifurcation parameter. Like in the previous section, the bifurcation diagrams of the system behaviour (Fig. 4a) and Lyapunov exponents (Fig. 4b) have been obtained for a single oscillator. A chaotic character of excitation causes an irregular motion of the oscillator in the entire range of the bifurcation coefficient (Fig. 4a). Fig. 4b illustrates how to vary the values of the two largest Lyapunov exponents of the single system (Eqs. (4a) and (4b)) versus the bifurcation coefficient h. The horizontal line at the top of the picture represents the largest constant ELE and the sloping line below shows the largest RLE. The comparison of this picture with the bifurcation diagram showing a distance between the trajectories (Fig. 4c) of the augmented system (Eqs. (4a)–(4d)) shows that *synchronization* appears when the largest RLE becomes negative in spite of the fact that the motion of the system is still chaotic (Fig. 4a). This fact leads to the conclusion that the appearance of synchronization is an effect of the disappearance of the sensitivity of the system response to the initial condition as a result of the parameter change. In other words, if the synchronization takes place, then the response of the system has a regular nature (negative RLEs—a stable sub-attractor in the response subspace) in spite of the observed chaotic behaviour of the system, which is caused by chaotic driving only.

3.2. Irregular discontinuous driving

In the next numerical example the time evolution of dynamic excitation has the form of an irregular discontinuous signal. This signal is composed of two alternatively occurring harmonic functions and is under the control of another signal, which is generated by the dynamical system of the Duffing type, working in the chaotic range. The time evolution of excitation and the

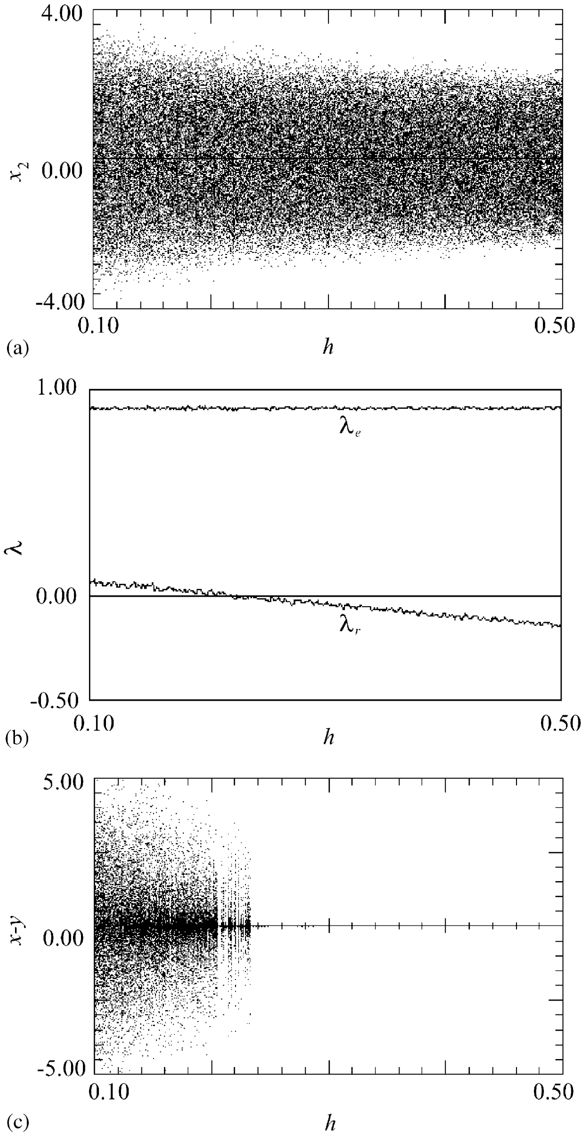

Fig. 4. Bifurcation diagrams of double oscillators system with chaotic excitation (Eqs. (4) and (5)) presenting: response of the system (a), the largest Lyapunov exponents (b) and distance between trajectories (c) versus bifurcation parameter h; $a = 1.00$, $q = 0.30$, $\delta = 10$, $r = 28$, $b = 8/3$.

mechanisms of signal generation are given by following equations:

$$\begin{aligned} \dot{u}_1 &= u_2, \\ \dot{u}_2 &= -u_1^3 - \delta u_2 + \beta \sin(\alpha t), \\ e(t) &= \sin(\gamma_1 t)(1 - \mathrm{sgn}(u_2)) + \cos(\gamma_2 t)(1 + \mathrm{sgn}(u_2)), \end{aligned} \tag{6}$$

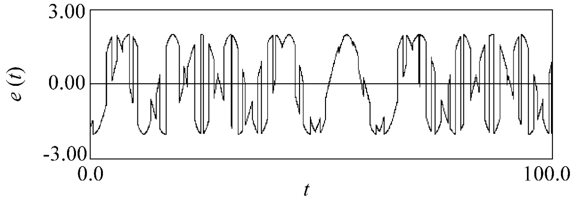

Fig. 5. Time evolution of the excitation—Eqs. (6).

where α, β and δ are the parameters of the Duffing system, and γ_1, γ_2 represent the frequencies of harmonic functions. The variable u_2 is the control signal and a function signum plays a role of a "switching function" between both harmonic signals, i.e., $e(t) = 2\sin(\gamma_1 t)$ for $u_2 < 0$, $e(t) = \sin(\gamma_1 t) + \cos(\gamma_2 t)$ for $u_2 = 0$, and $e(t) = 2\cos(\gamma_2 t)$ for $u_2 > 0$. The time diagram of the variable $e(t)$ shown in Fig. 5 illustrates a strongly discontinuous nature of excitation in this example.

The assumed character of driving causes the calculation of the Lyapunov exponents using the classical algorithm to be impossible. Therefore, the another method of estimation of the largest Lyapunov exponent, on the basis of synchronization of two coupled identical dynamical systems has been applied. This method exploits the phenomenon of a linear dependence between the largest Lyapunov exponent and the value of the coupling coefficient for which synchronization appears. In case under consideration, this method allows one to estimate the largest RLE. For practical application of the above-mentioned method, an uni-directionally negative feedback coupling between the oscillators under consideration has been introduced in the following form:

$$
\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= -ax_1^3 - hx_2 + qe(t),\\
\dot{y}_1 &= y_2 + d(x_1 - y_1),\\
\dot{y}_2 &= -ay_1^3 - hy_2 + qe(t) + d(x_2 - y_2),
\end{aligned}
\tag{7}
$$

where d is the coupling coefficient. The next step was a numerical research of the synchronization value of the parameter d which approximates the largest RLE (for a detailed description of the applied method—see Refs. [10–12]). The results of the estimation of this exponent are shown in Fig. 6b. The comparison of bifurcation diagrams shown in Figs. 6a and b confirms the existence of the connections between the largest RLE and the synchronization phenomenon which have been described in the previous section.

3.3. Randomly generated excitation

The excitation in the last example is a typical harmonic signal with the randomly changing amplitude $Q(t)$. Such a signal is generated according to the way given by the following equations:

$$\dot{\mu} = rnd, \tag{8a}$$

$$e(t) = \sin(\omega t)\sqrt{|\mu|}. \tag{8b}$$

Eq. (8a) describes the time evolution (Fig. 7) of the introduced variable μ which is under control of the random function rnd which returns a random number uniformly distributed in a set [−0.5,

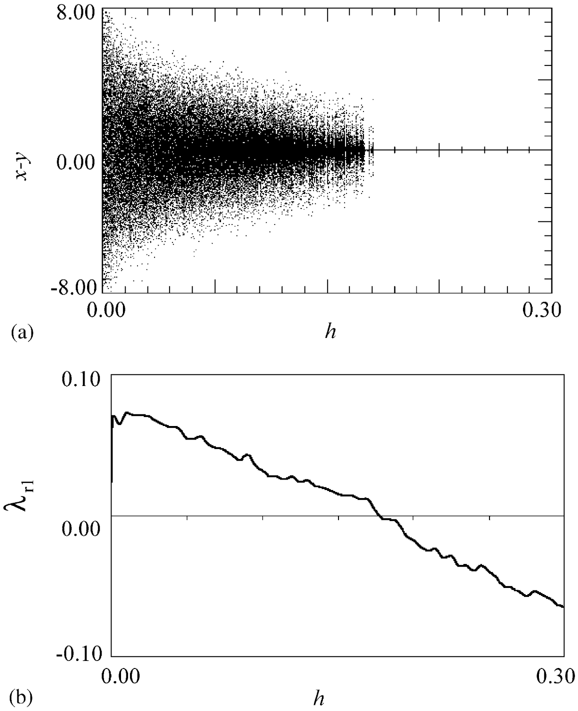

Fig. 6. Bifurcation diagrams of double oscillators system with irregular discontinuous driving (Eqs. (4) and (6)) presenting the distance between trajectories (a) and the largest RLE (b) versus bifurcation parameter h; $a = 1.00$,

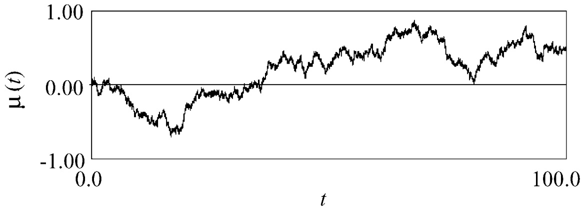

Fig. 7. Time course of stochastic function $\mu(t)$—Eq. (8a).

0.5]. An influence of this function is such that it causes random changes of the amplitude of external excitation ($Q(t) = q\sqrt{|\mu|}$ in Eq. (8b)) oscillating with the frequency ω.

The results of the synchronization process analysis in the system under consideration (Eqs. (4a)–(4d) and Eqs. (8a) and (8b)) are presented in Fig. 8. In order to compare the results, these numerical simulations have been performed simultaneously for two pairs of oscillators with different values of the damping rate, because it is impossible to repeat the same random process. Time diagrams of the distance between trajectories show that synchronization appears for a larger value of the damping parameter (Fig. 8a), like in the above-presented numerical examples with deterministic chaotic driving. But a lack of synchronization for a lower damping rate (Fig. 8b) is not an evidence for the fact that this state is stable. A random nature of the excitation can lead to temporary changes of the system response character. Hence, if the system response is working

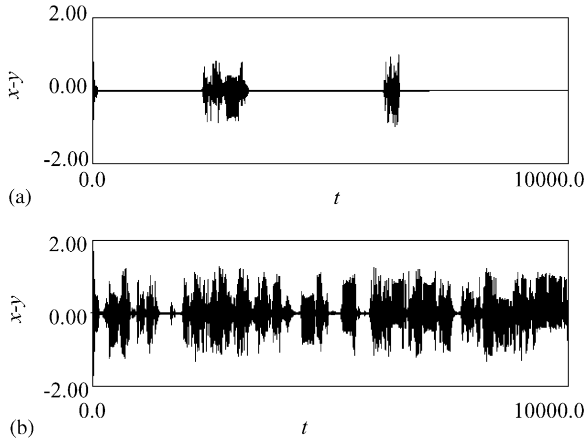

Fig. 8. Time diagrams representing the evolution of the distance between trajectories of double oscillators system driven by the same stochastic signal (Eqs. (4) and (8)) for different values of the damping coefficient: $h = 0.15$ (a) and $h = 0.05$ (b); $a = 10.00$, $q = 1.00$, $\omega = 1.00$.

long enough in the regular state (a period of its duration is determined by the period of the largest negative RLE occurrence), then synchronization can appear even for a small damping parameter. However, from the numerical analysis carried out follows that in spite of unpredictable dynamics of the oscillators, the synchronization tendency increases with the damping rate.

4. Remarks and conclusions

The above-presented theoretical considerations supported by the numerical simulations show, that a set of identical, separate mechanical oscillators forced by common irregular excitation tend to synchronize if all RLEs describing the evolution of each oscillator are negative.

The above fact is only the necessary, but not sufficient, condition of synchronization. To achieve the compliant synchronous motion of all oscillators with chaotic or random driving, they also have to start from the basin of attraction of the same sub-attractor in the response subspace. If initial conditions belong to different basins of attraction, then an additional coupling (even weak) between the oscillators is required to achieve synchronization. An occurrence of coexisting *sub-attractors* is more probable in many-degree-of-freedom or non-smooth systems.

The introduced term RLEs has a similar practical sense like *transverse Lyapunov exponents* (TLE) [2] or *sub-Lyapunov exponents* (SLE) [3], because negative values of these exponents are required for synchronization. However, RLEs can be calculated for a single oscillator on the contrary to TLEs and SLEs, where it is necessary to build a double oscillator system to determine its values. Negative RLEs generate sub-attractors in the response subspace and this situation leads to synchronization. Thus, to test the synchronization tendency of an arbitrary set of identical oscillators, it is enough to know the spectrum of Lyapunov exponents for one of them. On the other hand, if calculation of these exponents is not straightforward or even impossible, the appearance of synchronization informs about the regular nature of the system response even if the observed motion of the system is irregular due to chaotic or stochastic external excitation. The

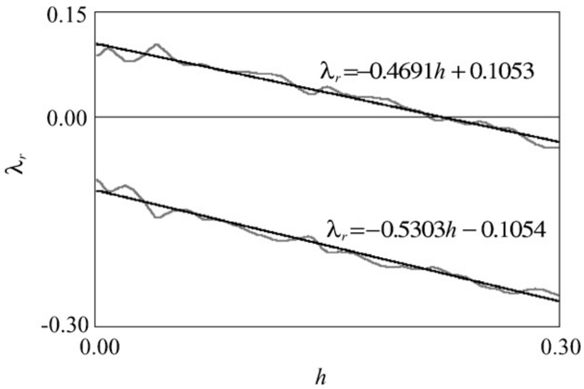

Fig. 9. Bifurcation diagram of double oscillators system with chaotic excitation (Eqs. (4) and (5)) presenting RLEs versus bifurcation parameter h and its trendlines; $a = 1.00$, $q = 0.30$, $\delta = 10$, $r = 28$, $b = 8/3$.

present considerations also lead to the two obvious conclusions: (i) synchronization of smooth linear oscillators appears always because in such a case RLEs are always negative, so the sub-attractor in the response subspace is stable, (ii) the presented mechanisms of synchronization is also valid for the case of periodic external driving.

The examples with chaotic excitation show an explicit border between synchronization and desynchronization (Figs. 4c and 6a). It is an effect of almost linear dependence between the largest RLE and the damping coefficient h shown in Figs. 4(b) and 6(b). To explain this phenomenon, let us consider a total divergence (D) of the system given by Eqs. (1), which is a sum of divergences associated with the excitation (D_e) and the response (D_r)—$D = D_e + D_r$. The excitation divergence D_e is independent of oscillator parameters. For the Duffing oscillator used in the numerical simulations (Eqs. (4)), the response divergence D_r is equal to the negative value of the damping rate ($D_r = -h$) or it can be considered as a sum of RLEs ($D_r = \lambda_{r1} + \lambda_{r2}$). Hence, the following equality is fulfilled:

$$\lambda_{r1} + \lambda_{r2} = -h. \tag{9}$$

Fig. 9 presents a more thorough analysis of both RLEs for the system considered in this paper (Eqs. (4)) with chaotic excitation (Eqs. (5)). It is clearly visible that Eq. (9) is fulfilled and almost parallel trendlines of both bifurcation courses inform that an influence of the increasing damping parameter on RLEs is distributed equally between them.

A similar linear dependence has been observed between the largest Lyapunov exponent and the synchronization value of the coupling coefficient (d) in the system of double identical dynamical subsystems coupled by the negative feedback mechanism [9–12]. Properties of such a coupling have been exploited for estimation of the largest RLE in case described in Section 3.2. Thus, the conclusion is that damping in mechanical systems with common driving play a role of the negative feedback coupling between these systems.

Such a linear influence of damping on the synchronization process has also important significance for practical considerations. Namely, it causes that large enough damping (negative RLEs) ensures the robust synchronization state vis-á-vis of perturbations, noises and even parameters mismatch. Obviously, a non-identical parameters of the oscillators make ideal

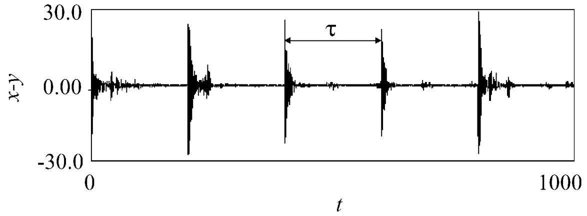

Fig. 10. The time diagram representing the evolution of the distance between trajectories of both oscillators driven by chaotic excitation (Eqs. (4) and (5)) with periodic perturbation ($\tau = 200$) and parameters mismatch ($\delta a/a = 0.03$); $h = 0.30$, $a = 1.00$, $q = 0.30$, $\delta = 10$, $r = 28$, $b = 8/3$.

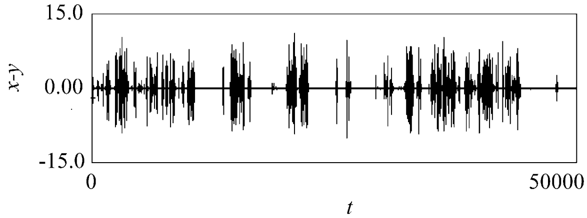

Fig. 11. Phenomenon of intermittency observed in double oscillators system with chaotic excitation (Eqs. (4) and (5)); $h = 0.215$, $a = 1.00$, $q = 0.30$, $\delta = 10$, $r = 28$, $b = 8/3$.

synchronization impossible, because in such a case both systems work on the different attractors. However, a small difference between these parameters can lead to the practical synchronization (mentioned in Section 1). An evidence for stability of practical synchronization in the system under consideration is shown in Fig. 10. It is the time diagram representing the evolution of the distance between trajectories of both oscillators driven by chaotic excitation (Eqs. (4) and (5)), where periodic perturbation of this distance (with period τ) and small difference of parameters (say, $a = a + \delta a$ in Eq. (4d)) are introduced. It can be observed, that in spite of perturbation and parameters mismatch ($\delta a/a = 0.03$), a "memory" about the synchronized state is retained and both systems tend toward practical synchronization after each perturbation. The same phenomenon causes the robust synchronization in the case of random excitation (Eqs. (4) and (8)), e.g., the increase of damping leads to the elimination of desynchronization bursts.

Another aspect of the synchronization phenomenon in the examples presented in the previous section (with chaotic excitation) is a process of transition between a chaotic and hyperchaotic motion. An appearance of synchronization shown in Figs. 4(c) and 6(a) informs of the transition from hyperchaotic behaviour of the system (characterized by two positive Lyapunov exponents) to the chaotic motion with one positive Lyapunov exponent without calculation of these exponents. A mechanism of such a transition in case under consideration is well-known phenomenon of on–off intermittency [13,14]. This phenomenon takes place in the neighbourhood of the synchronization value of the damping coefficient, shortly after the moment when the largest RLE becomes positive and is characterized by the temporarily bursting out of the invariant manifold $x = y$ and a relatively long evolution near this synchronization manifold (see Fig. 11).

Summing up, It can be stated that the phenomenon of synchronization described in this paper can occur in mechanical systems with external driving of irregular character. However, the numerical examples presented have been performed for the simplest system of two single-degree-of-freedom oscillators. Therefore, the following questions arise: How does such a process of synchronization by common excitation in many-degree-of-freedom mechanical oscillators or non-smooth systems proceed? Does common irregular driving lead to synchronization also in other kinds of dynamical systems described by differential equations (phase streams) as well as by difference equations (maps)? The answers to these questions and a more general description of the considered problem (in particular, for case of random driving) will be reported soon.

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