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# Stability regions of periodic trajectories of the manipulator motion

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**Abstract**

We present how to avoid dangerous situations that occur during a robot periodic motion and are caused by different kinds of vibrations. Theoretical analysis of stability regions and of the ways of inducing vibrations during a stability loss of periodic trajectories of the manipulator motion, based on the theory of nonlinear systems is developed. Based on the bifurcation diagrams and Poincare maps, an identification of stability areas has been carried out. To illustrate our method theoretically and numerically, a model of the *RRP*-type manipulator has been considered.


## 1. Introduction

In the case of stationary robots, the designed trajectories of a manipulator gripping device motion that result from the technological processes being carried out exhibit most often a character of periodic motion cycles. From the viewpoint of safety, an analysis of the behaviour of the dynamical system such as a manipulator that performs a periodic motion and is subjected to motion perturbations is an interesting issue. To this effect, the motion stability is understood as insensitivity of the trajectory of the dynamical system motion to motion perturbations. In this paper, an investigation of dynamical characteristics in the vicinity of the periodical trajectory, based on Poincare maps of variational equations, has been proposed. A Poincare map is understood as a discrete mapping in the form of $(n-1)$ of the dimensional space that divides the phase space into two subspaces and where $(n)$ is a dimension of the dynamical system. In order to achieve this goal, the equations of Poincare maps [12] have been written and they have been employed in nonlinear equations of perturbations. In the case of manipulators, an application of a Poincare map is convenient due to the fact that the position of the boundary cycle in the phase space is known. Because of this, it is relatively easy to define a Poincare map in the phase space. An analysis of stability regions makes the linearization of maps possible and constitutes an initial stage of an analysis of manipulator vibrations, that is to say, of manipulator bifurcation types. Owing to a possibility of occurrence of unpredictable vibrations due to a stability loss, this issue seems to be very important from the point of view of work safety. Sample stability regions of the manipulator, conditions under which a stability loss takes place and ways in which a stability loss of the manipulator periodic motion due to perturbations occurs have been presented.

Problems involved in an analysis of stability of periodic trajectories of dynamical systems are based on the Floquet and Lapunov theories [3,5,7,9,12]. The Floquet theory deals with linear differential equations with periodic coefficients. In turn, using the Lapunov theorem, we can state the stability of periodic solutions to the nonlinear equation on the basis of an analysis of the linear variational equation. The Lapunov theorem does not, however, inform what new

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solutions appear or disappear and what their stability is like, i.e., it does not characterise a bifurcation type. It follows from the fact that this theorem employs an analysis of the linearized equation. In order to determine a type of bifurcation, thus it is indispensable to investigate the nonlinear issue in the vicinity of the parameter value that corresponds to the boundary value of Floquet multiplicators. In typical cases, the investigation of behaviour of Floquet multiplicators that are obtained from the linear analysis allows for determination of a type of bifurcation of the periodic orbit. In other words, in order to identify a bifurcation type, it is sufficient to consider the forms of solutions to the linear variational equation that correspond to the transition of Floquet multiplicators through an elementary circle. As an example, methods of mechanical systems stability analysis based on different conditions can be found in [1,6,10].

Our task is to develop a model of the manipulator, a model of the electric and mechanical drive and to determine critical values of parameters of the nonlinear model for which a change in stability, i.e., bifurcation, takes place. Both the regions of stability and kinds of vibrations that can occur during a perturbation of the manipulator periodic motion are interesting and important. On the basis of the manipulator variational equation and employing the theory of Poincare maps, it will be possible to determine the regions of stability of a periodic trajectory of the manipulator gripping device motion. An analysis of stability regions performed for an *MAR* manipulator is presented in Section 4.

## 2. Model of the manipulator and of electric and mechanical driving systems

To simplify the motion equations and to clarify the usage of the method, a model of the manipulator (Fig. 1) with rigid links has been assumed. In the mathematical model of the manipulator with the vector $n$ of generalised coordinates of links, the potential energy has been written as a sum of the potential energy of links, an object being manipulated and the flexibility of kinematic pairs. The potential energy of flexibility has been written as a function of resultant flexibility of rolling, three-dimensional and slow-speed kinematic pairs [11]. The energy of dissipation of the kinematic pair has been expressed by means of a resultant coefficient of energy dissipation [4] as a function of square relative velocity of kinematic pair links.

The electric and mechanical model of the driving system of the manipulator covers torsional flexibilities, viscous damping and resistance to friction in driving systems. It has been assumed in these considerations that each manipulator link is driven by an independent driving system (Fig. 2) and consists of an electric motor, a brake, a mechanical gear and driving shafts [13]. A stator of the driving motor of the $i$th driving system is connected with the $i$th $-1$ link, i.e., it is a part of this link. Energy losses due to mechanical clearances in driving units and gyroscopic effects between motors and manipulator links have been neglected.

A vector of generalised coordinates of the manipulator is expressed as

$$\bar{q}_m = [\bar{q}, \bar{q}^s]^{\mathrm{T}} \tag{1}$$

where $\bar{q}$ is a vector of generalised coordinates of manipulators links, whereas $\bar{q}^s$ is a vector of generalised coordinates of driving motors.

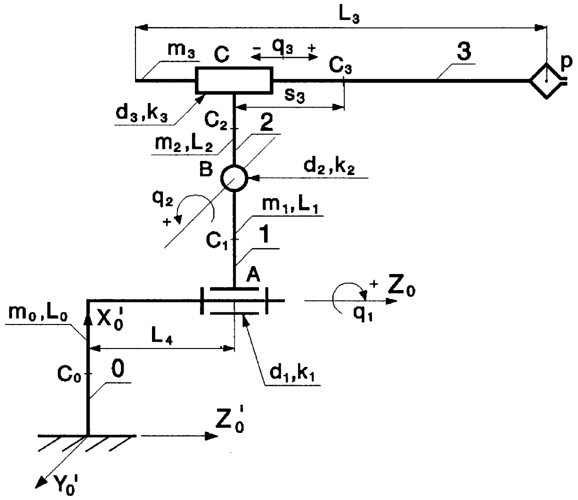



Fig. 1. Kinematics scheme of the manipulator.

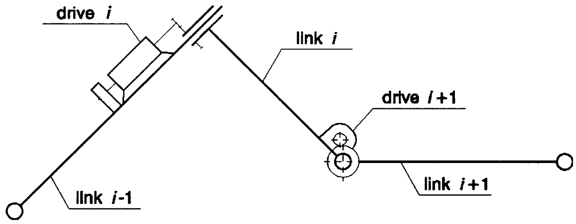



Fig. 2. Schematic diagram of the driving system of the manipulator link.

A rotor of every driving motor has its own frame of reference and an inertia tensor related to this system. In general, an inertia tensor of the driving motor rotor of the $i$th driving system is

$$I_{wi} = \begin{bmatrix} I_{xxi} & -I_{xyi} & -I_{xzi} \\ -I_{xyi} & I_{yyi} & -I_{yzi} \\ -I_{xzi} & -I_{yzi} & I_{zzi} \end{bmatrix} \tag{2}$$

where $I_{\alpha\beta i}$ are mass centrifugal moments of inertia of the $i$th rotor with respect to planes perpendicular to the axes $\alpha$ and $\beta$, $\alpha = \{x, y, z\}$, $\beta = \{x, y, z\}$ and $\alpha \neq \beta$, whereas $I_{\alpha\alpha i}$ are mass moments of inertia of the $i$th rotor with respect to main axes of inertia; $\alpha = \{x, y, z\}$.

The kinetic energy of the $i$th manipulator link, which is a sum of the kinetic energy of the link, the kinetic energy of the rotor of the driving motor of the $i$th driving system, and the kinetic energy of rotating elements of the $i$th system of power transfer, is equal to

$$E_{ki} = \tfrac{1}{2} \cdot \dot{q}_i^{\mathrm{T}} \cdot D(q_i) \cdot \dot{q}_i + \tfrac{1}{2} \cdot (\dot{q}_i^s)^{\mathrm{T}} \cdot I_{wi} \cdot \dot{q}_i^s + \tfrac{1}{2} \cdot (\dot{q}_i^s)^{\mathrm{T}} \cdot I_{ni} \cdot \dot{q}_i^s \tag{3}$$

where $D(q_i)$ is a tensor of inertia of the $i$th link along with the rotor mass and the elements of the $i+1$th and $i$th system of power transfer assigned to the link $i$, as well as of the electric motor stator of the $i+1$th driving system being a part of the link, according to Fig. 2. In general, the kinetic energy of the manipulator equals to

$$E_k = \tfrac{1}{2} \cdot [\dot{q}^{\mathrm{T}} \cdot D(q) \cdot \dot{q} + (\dot{q}^s)^{\mathrm{T}} \cdot I_{zr} \cdot \dot{q}^s] \tag{4}$$

where $D(q)$—matrix of inertia of the manipulator links $(n \times n)$; $I_w$, $I_n$—matrix of inertia of rotors of driving motors and shafts and rotating elements of reduction gears and brakes, reduced to the axis of generalised coordinates, correspondingly, $(n \times n)$; $I_{zr} = I_w + I_n$—moment of inertia of rotors of driving motors, power transfer shafts, rotating elements of reductions gears and brakes, reduced to the axes of respective generalised coordinates. The tensor of inertia reduced to the axis of the generalised coordinate $q_i$ of the $i$th link, of power transfer shafts and rotating elements of reduction gears and breaks can be expressed as

$$I_n = \mathrm{diag}[I_{n1}, \ldots, I_{nn}] \tag{5}$$

where $I_{n1}$, $I_{nn}$ depend on the assumed design of the systems of power transfer. The reduced moment of inertia of the driving system of the $i$th degree of freedom for the start-up is equal to

$$I_{zr} = \mathrm{diag}\left[\eta_{1s} \cdot n_1^2 \cdot I_{zz1} + \eta_{1s} \cdot \sum^{k} (n_{1k} \cdot I_{n1}^k), \ldots, \eta_{ns} \cdot n_n^2 \cdot I_{zzn} + \eta_{ns} \cdot \sum^{k} (\eta_{nk} \cdot I_{nn}^K)\right] \tag{6}$$

where $\eta_{ik}$—mechanical efficiency corresponding to the $k$th element of the $i$th drive; $\eta_{is}$—mechanical efficiency of the driving motor of the $i$th driving system; $I_{ni}^k$—moment of inertia of the $k$th element of the driving system of the $i$th degree of freedom, reduced to the axis of the generalised coordinate of the $i$th link. In the case of braking, we have

$$I_{zr} = \mathrm{diag}\left[\frac{n_1^2 \cdot I_{zz1}}{\eta_{1s}} + \sum^{k} \frac{I_{n1}^k}{\eta_{1s} \cdot \eta_{1k}}, \ldots, \frac{n_n^2 \cdot I_{zzn}}{\eta_{ns}} + \sum^{k} \frac{I_{nn}^k}{\eta_{ns} \cdot \eta_{nk}}\right] \tag{7}$$

The potential energy of the manipulator has been expressed as a sum of the potential energy of links, an object being manipulated, elements of power transfer systems and flexibility of kinematic pairs. The potential energy of flexibility of driving systems is described by a matrix of resultant torsional stiffnesses of power transfer systems that have been reduced to generalised coordinates of manipulator links [11]. This energy described as a function of generalised coordinates of links and rotors of motors reflects resultant matrices of stiffness of individual power transfer systems that are reduced to their corresponding generalised coordinates of the manipulator.

Thus, the potential energy of elements of power transfer systems is equal to

$$E_{\mathrm{p}}(\bar{q}) = \sum_{i=1}^{n}\sum_{j=1}^{l} E_{\mathrm{p}}^{n}ij(T_{1i-1}, T_{1i}) = \sum_{i=1}^{n}\sum_{j=1}^{l_1} E_{\mathrm{p}}^{n}ij(T_{1i-1}) + \sum_{i=1}^{n}\sum_{j=l_1+1}^{l=l_1+l_2} E_{\mathrm{p}}^{n}ij(T_{1i}) \tag{8}$$

where $E_{\mathrm{p}}^{n}ij$—potential energy of the $j$th element of the $i$th manipulator driving system; $l$—number of elements of the $i$th driving system, whose potential energy is considered in the equation of the manipulator potential energy; $ij$ denotes the $j$th element of the $i$th driving system; $l_1, l_2$—number of elements of the driving system of the link $i$ assigned to the link $i-1$ and $i$, respectively; $T_{1i-1}, T_{i1}$—matrices of transformations according to the Denavit–Harterberg notation. Hence, the Lagrange equations of the manipulator, reduced to the axis of the generalised coordinate $q_i$ of the $i$th link, assume the form

$$\begin{aligned} &D(q)\cdot\ddot{q} + C(q,\dot{q}) + K(q - q^{s}) = 0 \\ &I_{zr}\cdot\ddot{q}^{s} - K(q - q^{s}) = Q_1 \end{aligned} \tag{9}$$

where $D(q)$—matrix of inertia of the manipulator $(n \times n)$; $C(q, \dot{q})$—matrix of Coriolis forces, centrifugal forces and gravitational moments $(n \times n)$; $K$—diagonal matrix of reduced stiffnesses of driving systems $(n \times n)$; $I_{zr}$—diagonal matrix of reduced inertias of driving systems; $Q_1$—vector of driving quantities, reduced to the link axle.

It has been assumed that viscous friction in the driving system is a sum of viscous friction in the driving motor and viscous friction in the driving system, reduced to the axle of the driving motor. Having reduced the equations to the driving motor axle, we obtain

$$I_{zr}^{*}\cdot\ddot{\theta} + N^{-1}\cdot K\cdot(N^{-1}\cdot\theta - q) + B\cdot\dot{\theta} = Q_2 \tag{10}$$

where $N$—diagonal matrix of reduction gear ratios of driving systems $(n \times n)$; $I_{zr}^{*} = I_{zr}/N^2$—diagonal matrix of inertia of driving systems of the manipulator, reduced to driving axles of motors $(n \times n)$; $\ddot{\theta}$—vector of angular accelerations of rotors of driving motors $(n \times 1)$; $Q_2$—vector of driving quantities of links, reduced to generalised coordinates of rotors of driving motors $(n \times 1)$; $B$—diagonal matrix of viscous damping in driving systems $(n \times n)$ that has been expressed as follows

$$B = \mathrm{diag}\left[f_{w,w1} + \sum_{l=1}^{w^i} f_{w,u1}^{l}, f_{w,wi} + \sum_{l=1}^{w^i} f_{w,ui}^{l}, \ldots, f_{w,wn} + \sum_{l=1}^{w^i} f_{w,un}^{l}\right] \tag{11}$$

where $w^i$—number of elements of the $i$th driving system that are considered in determination of viscous friction; $f_{w,wi}$—coefficient of viscous friction in the $i$th driving motor; $\sum_{l=1}^{w^i} f_{w,un}^{l}$—sum of coefficients of viscous damping of individual elements of the power transfer system, reduced to driving axles of the motor.

Generally, the equations of motion of the manipulator in which electric and mechanical driving systems are taken into account assume the following form

$$[M]\cdot[\ddot{q}_m] + [C]\cdot[\dot{q}_m] + [K]\cdot[q_m] + [G] = [Q] \tag{12}$$

where $[M]$—matrix of inertia of the manipulator, composed of the moment of inertia of links, an object being manipulated and moments of inertia of driving systems that are reduced to the axis of the $i$th generalised coordinate; $[C]$—matrix of effects of gyroscopic forces, centrifugal forces and energy dissipation; $[K]$—matrix of stiffness; $[G]$—column matrix of gravity forces; $[Q]$—column matrix of driving quantities; $[q_m]$—column matrix of vectors of generalised coordinates of the manipulator and driving systems.

### 2.1. Trajectory of motion, kinematics and dynamics of the manipulator

A selection of the trajectory of motion of the manipulator gripping device, Fig. 3, has followed from the necessity of ensuring the continuity of components of vectors of velocity and acceleration of the point $P$ of the gripping device along the whole trajectory of its motion and from the fact that stops and start-ups of the manipulator have been taken into consideration, Fig. 5. Additionally, the assumed trajectory shows a possibility of occurrence of bifurcation of the motion trajectory (Figs. 7 and 8). An analysis of kinematics and dynamics that has consisted in solving reverse tasks makes it possible to define sets of configuration coordinates of the manipulator for which the assigned positions and

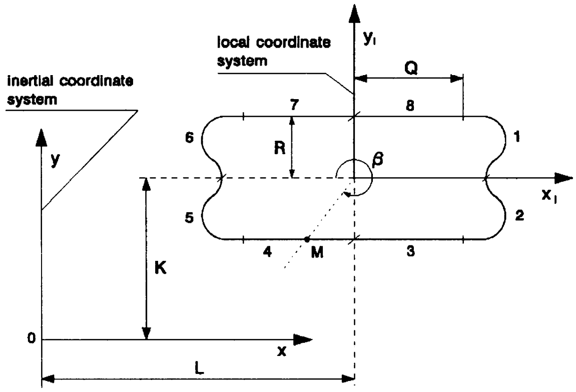



Fig. 3. Geometrical parameters of the trajectory of motion of the manipulator gripping device.

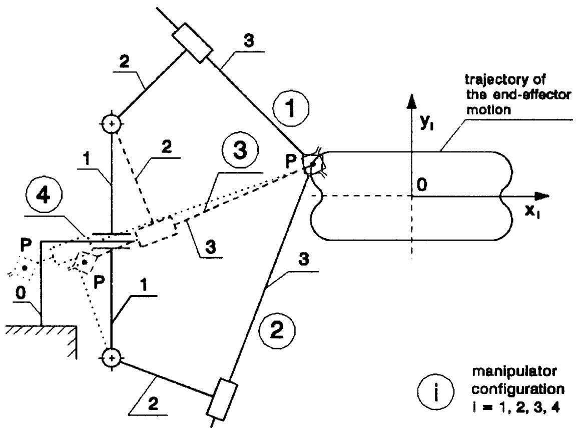



Fig. 4. Mathematical solution to the equations of manipulator positions.

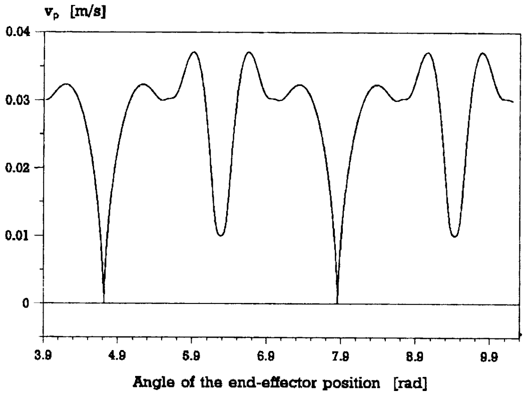
orientations of the robot working link can be obtained and to determine the driving quantities of individual degrees of freedom as a function of parameters of the trajectory of the gripping device motion, Figs. 7 and 8.

The Denavit–Hartenberg notation has been used to describe the kinematics. Four configurations of the manipulator that allow for any flat, vertical trajectory of motion of the gripping device have been found. Two of them are possible for the assumed kinematic structure of the manipulator. The manipulator configurations that permit the assumed trajectory are presented in Fig. 4. A numerical algorithm that solves a simple task of the dynamics of a manipulator with gears is to be found in [4,8].

Figs. 5–8 have been made for the calculation initial angle of the manipulator gripping device position on the motion trajectory $\beta_0(t = 0) = 225°$ (Fig. 3) where $\dot{\beta}_0 = \omega_0 = 0.3$ rad/s is an angular velocity of the motion of the calculation parameter $\beta_0$ along the gripping device motion trajectory with respect to the beginning of the frame of reference $X_1 Y_1 0$. The angle $\beta_0$ is a calculation parameter (uniformly variable in time) of the angle of the position of the manipulator gripping device centre on the motion trajectory in the local frame of reference $X_1 Y_1$ of the trajectory. This parameter allows for determination of the angle of the position of the point $P$ of the gripping device on the motion trajectory and its angular velocity (Fig. 6) for control systems. The parameter $\beta_0$ is useful for control systems.

## 3. Stability of periodic trajectories of the manipulator

The issue of stability of a periodic motion becomes important when the gripping device motion becomes unstable for some parameters of the manipulator model. Let us assume that the vector of generalised coordinates $\bar{q}(t) = [q_{01}(t), \ldots, q_{0n}(t)]^{\mathrm{T}}$ is a periodic solution to the equation of the manipulator motion and let us perturb this solution. A



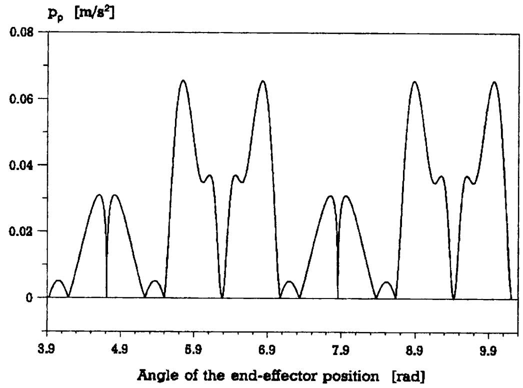




Fig. 5. Resultant velocity and acceleration of the point $P$ of the gripping device versus its position on the trajectory of motion.

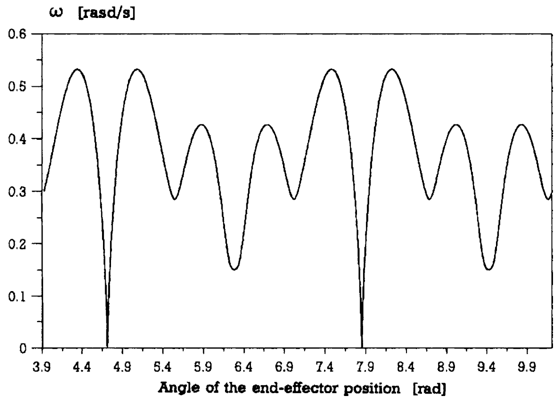



Fig. 6. Angular velocity of the gripping device motion versus time.

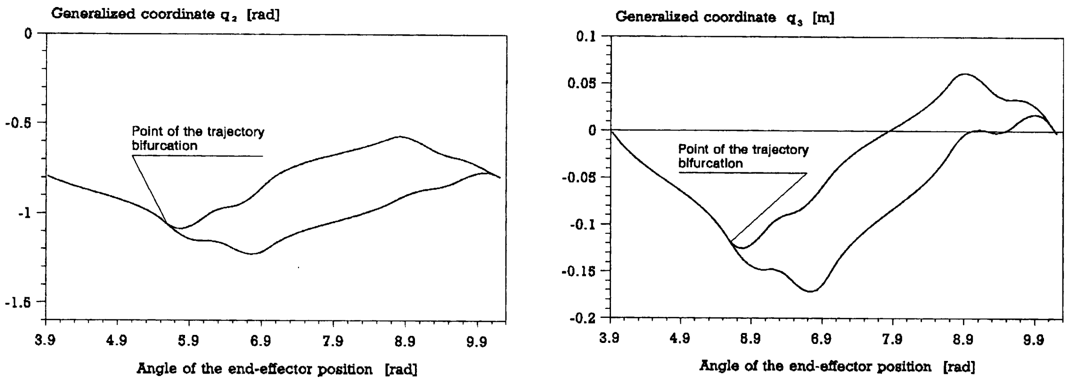



Fig. 7. Generalised coordinates $q_2$, $q_3$ as a function of the angle of the position of the point $P$ of the gripping device on the motion trajectory. A bifurcation point of the trajectory of generalised coordinates.

perturbation of the manipulator periodic solution is to be understood as a perturbation of the $i$th generalised coordinate/coordinates that is/are determined from the conditions of the robot nominal motion. In the suggested way of stability analysis, variational equations and the theory of Poincare maps have been employed. An analysis of stability regions makes the linearization of maps possible and constitutes an initial stage of the analysis of manipulator vi-

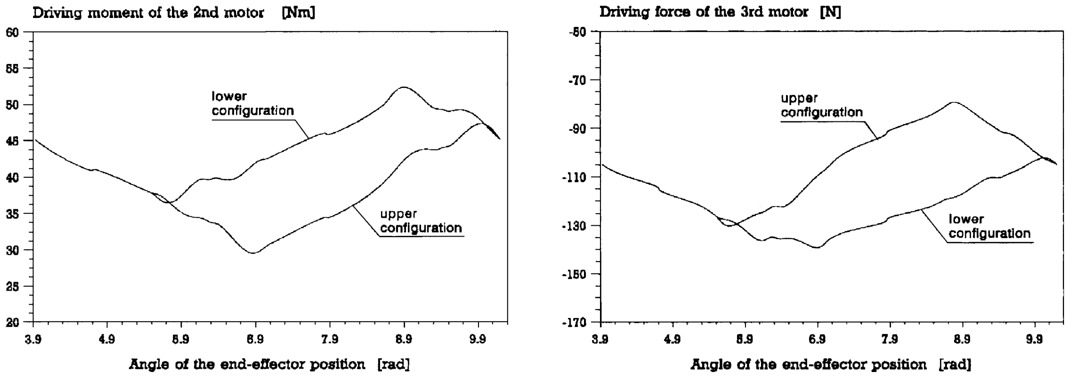



Fig. 8. Nominal driving quantities of electric motors of the periodic trajectory.

brations. A distance of the solution of the manipulator perturbed motion from the solution of the nominal motion, i.e., of the periodic motion, is defined by a variable $y(t)$.

Deflections of positions from the nominal motion and their time derivatives that appear in the mechanical system of the manipulator are compensated for by changes in values of driving torques of the nominal motion. An ability of mutual compensation of motion perturbations and changes in driving torques of individual degrees of freedom is connected with the assumed model of the control system of the robot motion. Let us write the vector of compensating driving quantities as $\overline{\Delta}$.

Substituting the vector of perturbations and compensation into the equations of motion of the manipulator, we obtain the variational equations

$$\dot{y} = A \cdot \bar{y} + B \cdot \overline{\Delta} + N(y, \dot{y}) \tag{13}$$

$\bar{y}, N \in R^{2n}$. Matrix $f$ includes nonlinear terms of equations of motion perturbations. A degree of nonlinearity of Eq. (13) depends on the form of the series expansion of trigonometric functions. As a result, we obtain differential equations of perturbations with periodic coefficients.

In the case under consideration of the manipulator during the motion of the second and third degree of freedom, whereas the first degree is stationary, we obtain [4]

$$\dot{y}_i = a_{ij}y_j + b_{in}\Delta_n + c_{2,ijk}y_jy_k + d_{2,ijm}y_j\dot{y}_m + c_{3,ijkl}y_jy_ky_l + d_{3,ijkm}y_jy_k\dot{y}_m \tag{14}$$

where $i, j, k, l = 1, \dots, 4$; $j \leqslant k \leqslant l$, $m = 2, 4$; $n = 2, 3$; $\Delta_n$—compensating drive; $a_{ij}$, $b_{in}$—linear part of the equations of motion; $c_{2,ijk}$, $d_{2,ijm}$, $c_{3,ijkl}$, $d_{3,ijkm}$—matrices of nonlinearity of the equations of motion. The matrices $a_{ij}$, $b_{in}$, $c_{2,ijk}$, $d_{2,ijm}$, $c_{3,ijkl}$, $d_{3,ijkm}$ depend on the nominal motion of the manipulator. The compensation vector is a set of parameters of the control system, owing to which compensation of deflections of the gripping device positions is possible. These parameters reflect a compensating driving quantity.

$$\overline{\Delta} = [\text{control system } 1, \dots, \text{control system } n]^{\mathrm{T}} \tag{15}$$

In general, the vector of parameters that have to be investigated as far as their effect on the motion stability is concerned can be presented as follows

$$\bar{\varepsilon} = [\text{kinematics of gripping device motion, control parameters, trajectory parameters}]^{\mathrm{T}} \tag{16}$$

The problem of stability of the periodic motion of the gripping device is formulated as an analysis of stability of the matrix of equations of the perturbed motion as a function of $\bar{\varepsilon}$.

$$\bar{y} = 0 \tag{17}$$

The Poincare map in the vicinity of (17) is defined as

$$\Pi[\bar{y} + y(t = 0)] = y[y(t = 0), T] \tag{18}$$

where

$$y(t = 0) \tag{19}$$

are the initial conditions of the motion, and $T$ is period [5].

In order to describe the stability of the periodic solution more precisely, relationship (18) in the vicinity of (17) has been expanded in the following series

$$\Pi[y(t=0)] = \frac{\partial y}{\partial y(t=0)} \cdot y(t=0) + \frac{1}{2} \cdot \frac{\partial^2 y}{\partial y^2(t=0)} \cdot (y(t=0), y(t=0)) + \frac{1}{6} \cdot \frac{\partial^3 y}{\partial y^3(t=0)} \cdot (y(t=0), y(t=0), y(t=0)) + \cdots \tag{20}$$

Subsequent partial derivatives in Eq. (20) are calculated as a function of the period of the manipulator motion as partial derivatives of variational equations (13) written in the form

$$\dot{y} = f(y, t) \tag{21}$$

If we assume a close neighbourhood of column matrix (17), then a linear analysis of stability in the vicinity of (17) can be assumed as an approximation. Thus, we deal with a linearized discrete mapping in the form of a Poincare map in the vicinity of the periodic trajectory. The analysis of stability of column matrix (17) is reduced then to the analysis of eigenvalues

$$E(\Lambda) = \frac{\partial y}{\partial y(t=0)} \tag{22}$$

In order to perform this analysis, we have to generate a matrix $A(t)$ composed of derivatives of the equations of the perturbed motion as a function of parameters of perturbations in the following form

$$A(t) = \frac{\partial f_i(y_{i0})}{\partial y_i} \tag{23}$$

for the initial conditions (19). The eigenvalues of the perturbation equations in the vicinity of initial conditions (19) can be determined from the determinant

$$\det[A(t) - \overline{\Lambda} \cdot I] = 0 \tag{24}$$

where $\overline{\Lambda}$—vector of eigenvalues; $I$—unit matrix.

The roots of Eq. (24), called characteristic multipliers of the periodic solution, determine stability of the periodic solution of the manipulator. Transforming Eq. (24), we obtain in the vicinity of initial conditions (19)

$$\Lambda^4 + A_1 \cdot \Lambda^3 + A_2 \cdot \Lambda^2 + A_3 \cdot \Lambda + A_4 = 0 \tag{25}$$

where

$$A_1 = -\frac{\partial f_2}{\partial y_2} - \frac{\partial f_4}{\partial y_4} \qquad A_2 = \frac{\partial f_2}{\partial y_2} \cdot \frac{\partial f_4}{\partial y_4} - \frac{\partial f_2}{\partial y_4} \cdot \frac{\partial f_4}{\partial y_2} - \frac{\partial f_4}{\partial y_3} - \frac{\partial f_2}{\partial y_1}$$

$$A_3 = \frac{\partial f_2}{\partial y_2} \cdot \frac{\partial f_4}{\partial y_3} - \frac{\partial f_2}{\partial y_3} \cdot \frac{\partial f_4}{\partial y_2} + \frac{\partial f_2}{\partial y_1} \cdot \frac{\partial f_4}{\partial y_4} - \frac{\partial f_2}{\partial y_4} \cdot \frac{\partial f_4}{\partial y_1} \qquad A_4 = \frac{\partial f_2}{\partial y_1} \cdot \frac{\partial f_4}{\partial y_3} - \frac{\partial f_2}{\partial y_3} \cdot \frac{\partial f_4}{\partial y_1}$$

The knowledge of characteristic multipliers makes it possible to obtain a spectrum of Lapunov exponents [5,12] of the periodic trajectory

$$\lambda_i = \frac{\ln|\Lambda_i|}{T} \tag{26}$$

where $T$ is a period of the gripping device motion along the trajectory of its motion. The periodic solution is stable if values of Lapunov exponents are negative or equal to zero.

## 4. Determination of stability regions of the *MAR* manipulator

The *MAR* robot manipulator (Fig. 1) whose main data are presented in Table 1, has been subjected to a sample numerical analysis in order to determine stability regions. The eigenvalues are calculated for varying parameters of velocity of the gripping device motion along the motion trajectory and for control parameters. The stability of matrix (17) is determined by absolute values of eigenvalues of Eq. (22), described with respect to 1 [5,12]. Thus, matrix (17) is

Table 1
Physical and geometrical data of the manipulator under analysis

| Number of the manipulator link | 1 | 2 | 3 |
|---|---|---|---|
| Link mass $m_i$ (kg) | 12.7 | 12.7 | 15 |
| Link length $L_i$ (m) | 0.18 | 0.18 | 0.42 |
| *Position of the centre of gravity in corresponding local coordinate system (m)* | | | |
| $x$ | $-L_1/2$ | $-L_2/2$ | 0 |
| $y$ | 0 | 0 | 0 |
| $z$ | 0 | 0 | $-L_3/2$ |
| $s_3 = 0.18$ m | | | |

asymptotically stable when all eigenvalues of Eq. (22) have their absolute values smaller than 1. If at least one eigenvalue has an absolute value higher than 1, then the motion of the system is unstable. In the case when all eigenvalues have their absolute values smaller than 1 or equal to 1, then the system is on the stability threshold.

Below, a few sample diagrams of stability regions for the angle $\beta = 315°$, which is the angle of the gripping device position on its motion trajectory (Fig. 3) and for the coefficients of damping and flexibility in kinematic pairs equal to zero, are shown. The data concerning the motion trajectory of the gripping device are presented in Table 2.

A linear control version has been considered, where the coefficient $U_s$ is a controlled variable and the coefficients $a$, $b$ are the coefficients of proportionality. The coefficient $a$ is connected with second drive system instead coefficient $b$ with third drive system.

In Fig. 9, a stability region as a function of the control coefficient $U_s$ and the angular velocity of the gripping device motion along its trajectory for the coefficients $a = -45.6$, $b = -0.003$ is depicted. One can see the ranges of parameters for which the system is stable. In turn, Figs. 10 and 11 show stability regions as a function of the coefficients $a$ and $b$ for the control coefficient $U_s = 1.51$ and the angular velocity equal to 0 and 0.1 rad/s, respectively. A decrease in the size of the stability region with an increase in the angular velocity of the gripping device motion along the trajectory of its

Table 2
Parameters of the manipulator gripping device motion

| Parameters of the gripping device motion trajectory | $Q$ (m) | $L$ (m) | $K$ (m) | $R$ (m) |
|---|---|---|---|---|
| – | 0.05 | 0.231 | 0.5 | 0.05 |

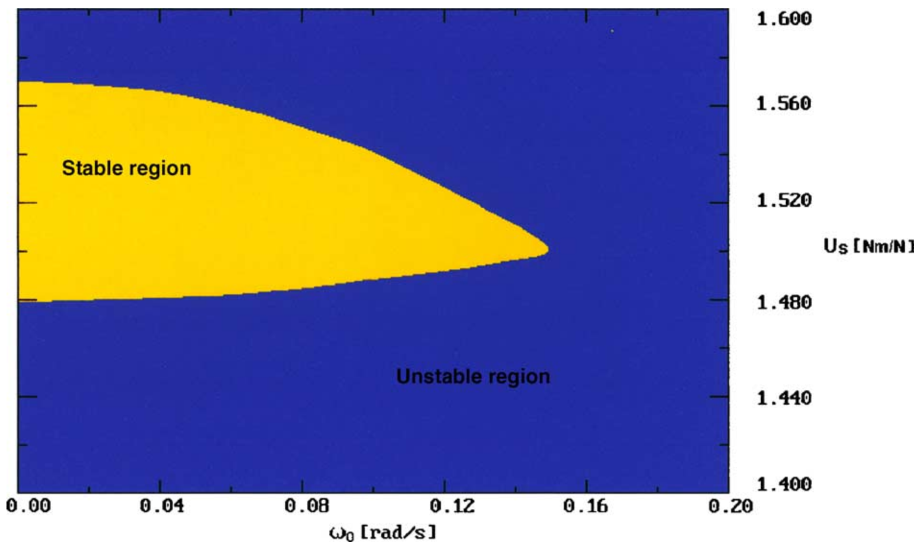



Fig. 9. Stability region as a function of the control variable $U_s$ and the kinematics of the gripping device motion. (Dark area: unstable region.)

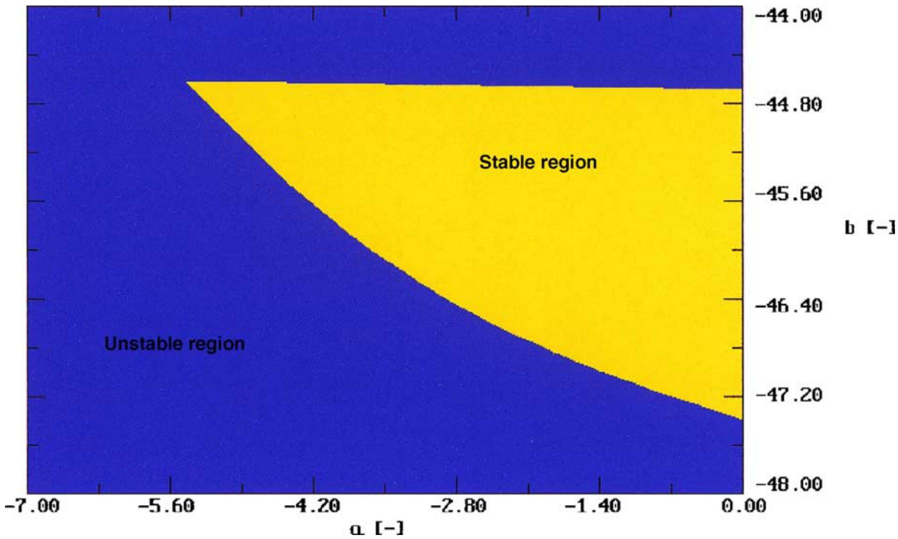



Fig. 10. Stability region as a function of proportionality coefficients in the control systems $a$ and $b$. (Dark area: unstable region.)

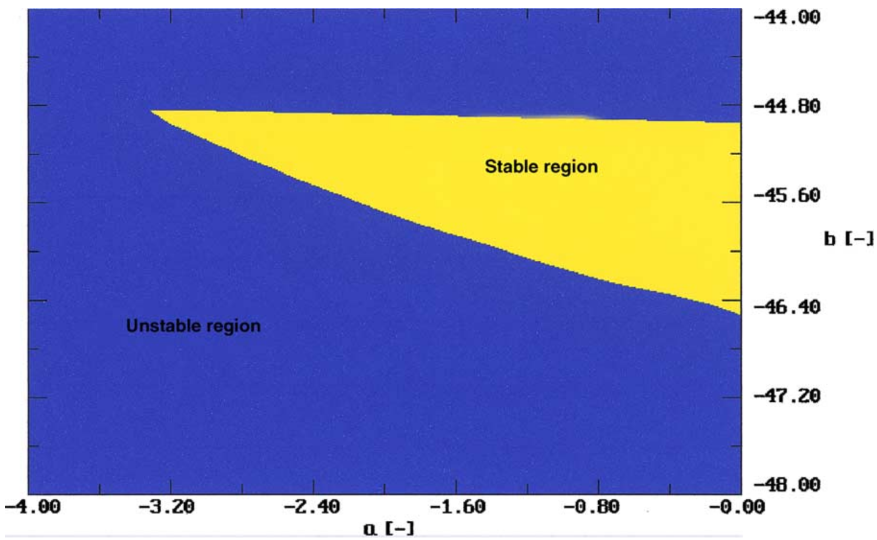



Fig. 11. Stability region as a function of proportionality coefficients in the control systems $a$ and $b$. (Dark area: unstable region.)

motion can be observed. In Fig. 12, we can see a stability region as a function of the quantity $L$ of the position of the motion trajectory (Fig. 3) and the velocity of motion of the gripping device along the trajectory of its motion. The following values of the control coefficients have been assumed: $U_s = 1.51$, $a = -45.6$, $b = -0.003$. An influence of the position of the trajectory of motion of the gripping device on the manipulator stability region is visible.

As a result of the theoretical analysis, it has been found that there is a possibility of occurrence of three types of bifurcation that are connected with a loss of stability of the manipulator motion. Vibrations that occur in the system accompany each kind of bifurcation. Because of damage that such phenomena can cause, we tend to eliminate a possibility of their occurrence through maintaining the operation of driving systems within ranges of a stable motion.

Figs. 13 and 14 present sample spectra of Lapunov exponents for the data included in Figs. 9 and 12, respectively. One can see stability and instability ranges of the manipulator as a function of various parameters of the manipulator.

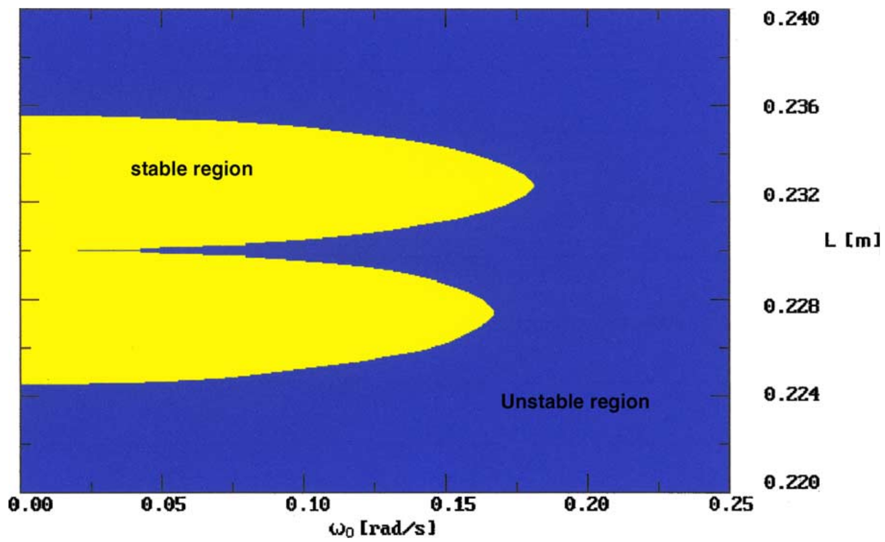



Fig. 12. Stability region as a function of the dimension $L$ of the position of the motion trajectory of the gripping device and the kinematics of its motion.

## 5. Conclusions

The analysis of stability regions of the periodic motion trajectory, has been carried out on the basis of Poincare maps for non-autonomous, nonlinear systems of differential equations. The presented analysis of the periodic trajectory stability makes it possible to analyse the regions of parameters in which the system operation is advantageous. A phenomenon of instability of motion of manipulator links shows a possibility of occurrence of unpredictable vibrations of individual robot links that can endanger the safety in the manipulator vicinity. The method can be used to investigate the manipulator stability for various trajectories of the gripping device motion, different parameters of control systems of driving systems and parameters of the manipulator gripping device motion along its motion trajectory.

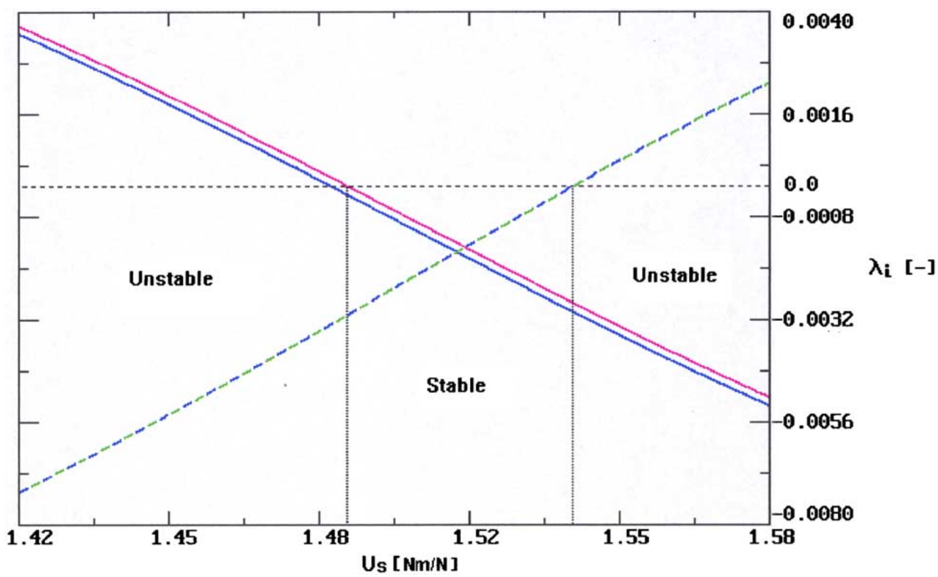



Fig. 13. Spectrum of Lapunov exponents for the data presented in Fig. 9 and $\omega_0 = 0.1$ rad/s.

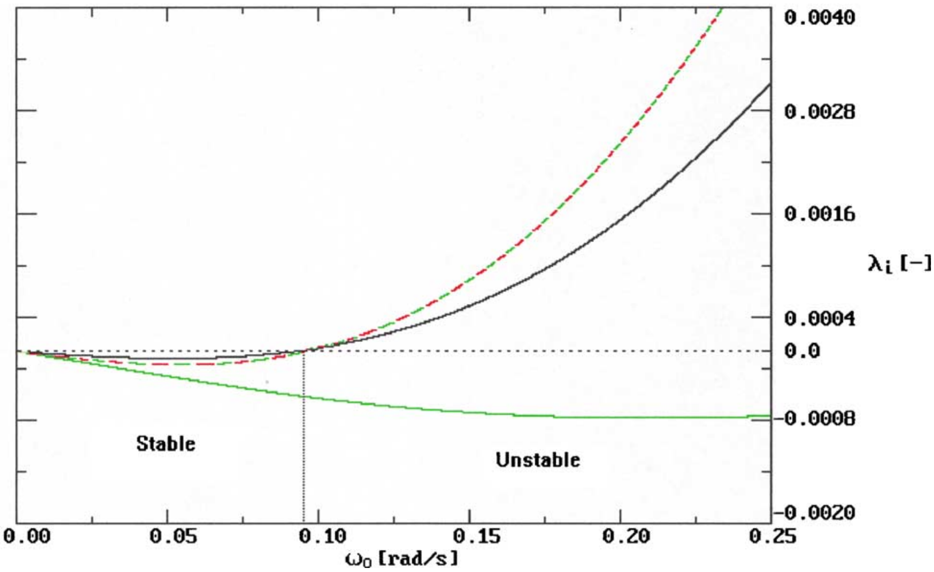



Fig. 14. Spectrum of Lapunov exponents for the data presented in Fig. 12 and $L = 0.225$ m.

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