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Chaos caused by non-reversible dry friction

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Abstract

In this short communication we investigate how the non-reversible dry friction characteristics will alter the non-linear responses of a simple mechanical oscillator. The presented approach is based on a certain mathematical description of the experimentally determined non-reversible dry friction characteristics, which causes chaotic and irregular motion of the studied system. A novelty of our idea is an introduction of the relative acceleration in description of the dry friction model.
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1. Introduction

For many years the topic of dry friction has been actively researched with many attempts to identify the causes of unwanted behavior such as squeal of car brakes, extensive wear of the cutting tools, and others. From the mathematical point of view dry friction problems are also cumbersome as the inclusion of the dynamics of dry fiction implicates appearance of the discontinuous differential equations, where the character of this discontinuity depends upon the friction character adopted. In general, there are many different types of dry friction models and it is crucial to appropriately chose one which suits best to the modelled problem. For example, if one considers dynamics of the system where the relative velocity practically remains constant, there is no need for sophisticated dry friction models and even the simplest one described by the Coulomb law will suffice. However in many cases the variation of the relative velocity is large and often the velocity changes its sign. In such situations the chosen model must account for the transition from static to dynamic friction and must provide a means of guiding the system through zero relative velocity. These types of mathematical models should be able to predict both phases of stick with a higher friction coefficient and slip where this coefficient is smaller. These are the reasons why systems with dry friction possess many different types of dynamical behavior, such as periodical, non-periodic, chaotic and sometimes even static responses [2,3,6–14].

A practical engineering approach, indebted to Coulomb simplifies the friction force to constant value directed opposite to the relative velocity of the contacting bodies. Such force can take two values with identical level but opposite sign only. Newer experiments show non-linear dependence on the contact velocity rather than the constant one. That was why most efforts were directed to built non-linear friction models and to determine differences in maximal values of the static and dynamic friction forces—see [4]. Another attempts to determine different types of friction characteristics showing dependencies on the relative acceleration on the contacting surface are so-called non-reversible friction characteristics [1,5,10,12].

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However, from a viewpoint of the experiment, even such non-linear approaches well approximate real character of friction force only for periodic responses of the considered system. In case of irregular, non-periodic motion of a non-linear friction oscillator, friction characteristics generated from experiments, often have a form of area filling curves, in co-ordinates friction force versus relative velocity [8]. This fact cannot be explained by measurement errors and rises a need to formulate a more general, universal friction model taking under consideration also the irregular motion of the system with dry friction. Therefore, in this brief letter we present a proposal of such a model, which has been elaborated on the basis of the known friction model [5] and the experimental results [8,12].

2. Dry friction modelling

We will use the experimental data obtained from the dry friction oscillator [12], which allows to induce conditions where the relative velocity changes its sign. There is a good deal of flexibility in varying frequency and amplitude of excitation, and combination of frictional materials. In this study we will be using the cleanest data which comes from experiments steel on teflon. To avoid the double counting of viscous forces, in Fig. 1, the viscous component in each dry friction force-relative velocity loop has been subtracted.

The above described dependence between friction characteristic reconstructed experimentally and a kind of observed motion has been induced us to new approach to the problem of dry friction modelling. According to our intention such a developed model of dry friction should have a universal character e.g. it well approximates real relation between friction force and relative velocity in case of irregular motion of the system and it simplifies to a form similar to known friction characteristics when motion is regular. This condition is fulfilled by the example of dry friction model presented below.

Other recent experimental research on the dry friction phenomenon carried out by authors [8] have led to the hypothesis, that temporary value of dynamic friction force is not only a function of relative velocity, but it also depends on the value (not only the sign as in classical non-reversible model) of the relative acceleration. If motion of the investigated system is of regular nature, i.e. has a periodic or multi-periodic attractor, then changes of relative velocity and acceleration repeat every period of system motion. According to the above hypothesis cycles of changes in friction coefficient values as relative velocity function are also repeatable. Harmonic or simple periodic motion leads to the well known approaches, where friction force is a function of relative velocity and sign of acceleration. In case of multi-periodic motion, more complicated form of such limit cycle, at which different traces of relative velocity and acceleration are realized, causes an appearance of additional lines in friction characteristic. In case of chaotic attractor there exist infinitely many unstable orbits lying on this attractor. As an effect there appear infinitely many possible traces, which fill an area of friction characteristics graph, as phase portrait of chaotic attractor fills some area of the phase space. Summing up, as the consequence of introducing the relative acceleration into the dry friction model, a closer relation between the friction characteristics and the system dynamics occurs, so this fact can be exploited in modelling of dry friction for non-linear systems.

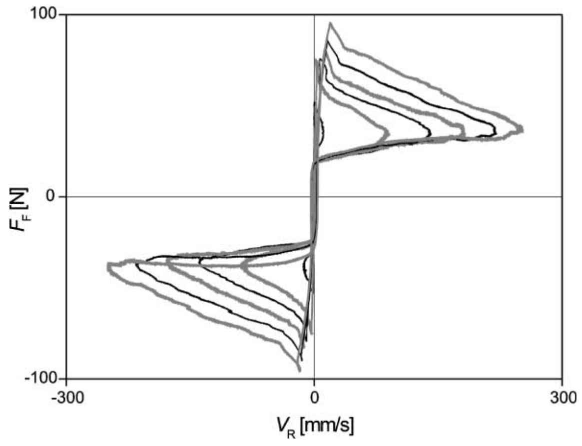

Fig. 1. Family of dry friction versus relative velocity curves.

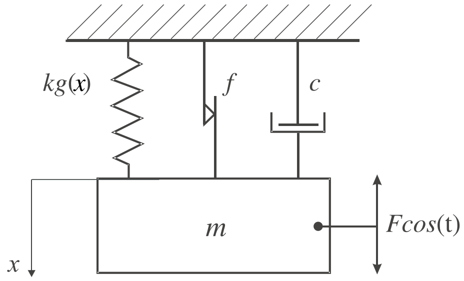

Fig. 2. Dry friction oscillator.

An example of such a modelling has been performed on the basis of non-reversible friction model [5]. The proposed novel model has a symmetrical non-reversible characteristics (Eq. (1a), with an auxiliary assumption that the parameter in the exponent is a function of the relative acceleration (Eq. (1b)) in contrast to primary model, where this coefficient is constant. Hence, the model is described with the following equations:

$$f = \begin{cases} f_u, & \operatorname{sgn}(\ddot{x}) > 0 \\ f_l, & \operatorname{sgn}(\ddot{x}) < 0 \end{cases} \rightarrow \begin{array}{l} f_u = f_d\left[1 + \frac{f_s - f_d}{f_d}\exp(-a(\ddot{x})|\dot{x}|)\right] \\ f_l = f_d\left[1 - \frac{f_s - f_d}{f_d}\exp(-a(\ddot{x})|\dot{x}|)\right] \end{array} \tag{1a}$$

$$a(\ddot{x}) = \frac{a_1}{|\ddot{x}| + a_2} \tag{1b}$$

where a_1, $a_2 > 0$ are constant parameters f_s, f_d—static and dynamic friction coefficient respectively.

To show universal character of the above presented model let us consider one-degree-of-freedom, mechanical oscillator to which a dry friction damper has been added (depicted in Fig. 2), where the model under consideration (Eq. (1)) has been applied in numerical simulations. The dynamics of this system can be described by a simple non-smooth second order differential equation

$$m\ddot{x} + kg(x)x + c\dot{x} + Nf\operatorname{sign}(\dot{x}) = F\cos(\omega t) \tag{2}$$

where: m—oscillator mass, $kg(x)$—spring stiffness, c—damping coefficient, ω—excitation frequency, F—amplitude of the excitation force, N—normal pressure force, f—friction function. Relative velocity of the friction surfaces is equal to the momentary velocity of the system $\dot{x}$.

After dividing both sides of Eq. (1) by mass and introducing: $x = x_1$, $\dot{x} = x_2$, $\alpha = k/m$, $2h = c/m$, $q = F/m$, $\varepsilon = N/m$, the analyzed system can be transformed to a set of two first order differential equations given below

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= q\cos(\omega t) - \alpha g(x_1) - 2hx_2 - \varepsilon f\operatorname{sign}(x_2), \end{aligned} \tag{3}$$

Universal nature of proposed method of dry friction modelling is presented in Fig. 2a and b, where friction characteristics for two different types of the oscillator under consideration and respective phase portraits of these systems are shown. It is clearly presented that for linear oscillator having periodic solution (Fig. 2a, $g(x) = x$), friction model possesses a shape predicted in mathematical description of the classical non-reversible model and contains two lines representing relative the acceleration and deceleration phases, respectively. In case of non-linear oscillator with chaotic attractor (Fig. 2b, $g(x) = x^3$) friction characteristic is an transformed image of this attractor according to the hypothesis presented above.

3. Concluding remarks

In this paper it has been shown, that the relative acceleration of sliding bodies used in the description of the dry friction model leads to the similarity between system attractor and friction characteristic. In other words, friction characteristics is a certain representation of the system dynamics. However, this is not a simple mapping, which shifts description of system dynamics from one co-ordinate system to the another one, because friction characteristic is an

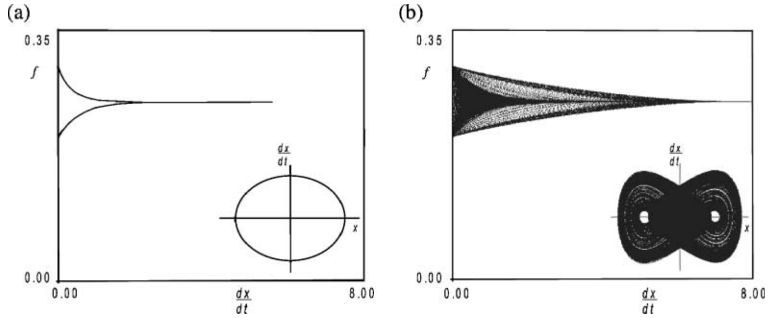

Fig. 3. Friction characteristics for the novel friction model and the respective phase portraits of the oscillator applied in numerical experiment for two cases: (a) linear friction oscillator—$g(x) = x$, $\alpha = 1.00$, $h = 0.05$, $q = 0.50$, $\eta = 1.00$, $\varepsilon = 0.5$, (b) non-linear friction oscillator—$g(x) = x^3$, $\alpha = 1.00$, $h = 0.05$, $q = 10.00$, $\eta = 1.00$, $\varepsilon = 0.5$; parameters of the friction model (Eq. (1)): $f_s = 0.30$, $f_d = 0.25$, $a_1 = 12$, $a_2 = 0.10$.

integral part of dynamical system model and also influences to its motion. On the basis of this property we propose a novel way of dry friction modelling. This approach has more general, universal nature, i.e. it develops a description of friction phenomenon on non-linear systems having irregular attractor but also it reduces to the known models of dry friction in case of regular motion. This fact is confirmed by the numerical simulations what is clearly visible in Fig. 3.

The model of dry friction presented in previous section only exemplifies the way of friction modelling and it is only the first step to formulate a novel model of dry friction for non-linear systems. Earlier numerical experiments carried out by authors have shown, that introduction of this model represented by Eq. (1), does not results in qualitative changes of motion character in comparison with another friction models (Readers are advised to see [7] for a comprehensive description of this comparison). Hence, from a viewpoint of engineering applications, where the dynamics is simple, it is sufficient to apply classical Coulomb law. However, our recent experiments show that even for our oscillator where $g(x)$ is non-linear, due to the contact stiffness only, the system can respond with an irregular stick-slip motion. Therefore there is a need for more comprehensive dry friction models, and this paper outlined one.

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