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Classification principles of types of mechanical systems with impacts – fundamental assumptions and rules

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Abstract

The way in which subsequent types of mechanical systems with impacts with n degrees of freedom arise and their classification are shown. The presentation of classification principles is a new compilation, according to the knowledge of the authors. The paper answers the question: how many types of systems with impacts exist in general and what these types are, and it leads to numerous conclusions, as well as shows directions of future investigations. Systems with one and two degrees of freedom are considered in detail. The models of systems under consideration are rigid bodies connected by means of, for instance, springs, which can perform a motion along a straight line without a possibility of rotations. For such systems, a complete spring–impact classification has been presented. A simple way of the notation of mechanical systems with impacts, consistent with the principles of the classification developed, has been proposed. The presented classification principles of types of mechanical systems with impacts are of fundamental importance in their designing processes.

Keywords: Basic spring system; Basic impact system; Apt combination; Inapt combination

1. Introduction

The investigations devoted to systems in which a phenomenon of nonsmoothness caused, for instance by impacts or dry friction, occurs, are becoming more and more important and more and more frequently analysed, despite the fact that their description by means of classical mathematical methods involves many difficulties. Lately, a survey of analytical and numerical methods for analysis of such systems has been published (Awrejcewicz and Lamarque, 2003). As these systems are the noncontinuous ones, chaotic motions that give a way to a thorough analysis in the field of the theory of bifurcation and chaos occur in them as well, apart from regular behaviour.

In the world literature, one can meet various mechanical systems with impacts (see, e.g., references). They are called impact oscillators (the literature published in English (Bishop, 1994)) or vibro-impact systems (the literature published in Russian (Babitskii, 1978)). In Fig. 1, a few schemes of mechanical systems with impacts that can be technically realised as models of impact vibration dampers, have been presented. The reader has probably noticed that they differ as far as the design of their component elements is concerned, which results in their various dynamical behaviour. The system shown in Fig. 1(a) is a body of a certain mass 1 connected with a frame by an elastic supporting structure. A second body of mass 2 is connected to it by means of a similar structure. A fender (symbol ⊤ turned by 90°) mounted in a fixed way is an additional element in system 1, thus under some properly assigned initial conditions, one-sided impacts can occur in this system. The system in Fig. 1(b) differs from the one in Fig. 1(a) as it has an additional fender. This time, two-sided impacts can occur in the system. A similar situation can occur in the system depicted in Fig. 1(c), however in this case a body of mass 2 does not have any supporting structure. Such

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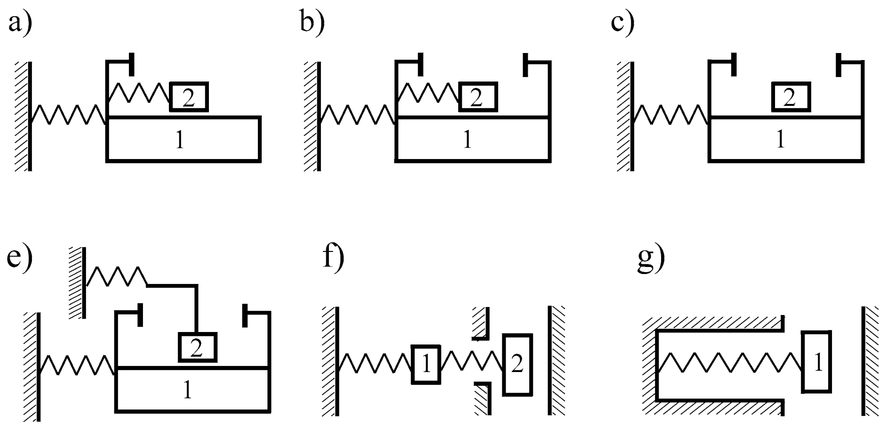


Fig. 1. Schemes of various types of impact vibration dampers.

structures are not present for bodies of mass 2 and 3 in Fig. 1(d), either. Fig. 1(e) shows another possibility of the solution of the support of bodies. Each body (of mass 1 and 2, respectively) has an independent structure that supports it with the frame. A body of mass 1 is equipped with two, fixed fenders that make two-sided impacts possible in the system. For all the above-described cases (Fig. 1(a)–(e)), the common characteristic feature lies in the fact that fenders that are mounted to one body in a fixed way displace with it. Slightly different cases are represented by the systems in Fig. 1(f) (two degrees of freedom) and Fig. 1(g) (one degree of freedom). In both the systems, two-sided impacts take place, but this time they are impacts on the frame.

While analysing the studies devoted to mechanical systems with impacts, one can state that researchers have focused on systems that differ in various aspects, namely:

(A) number of degrees of freedom – systems with one degree of freedom (e.g., Awrejcewicz and Lamarque, 2003; Babitskii, 1978; Blazejczyk-Okolewska et al., 1999; Cempel, 1970; Peterka, 1971, 1981; Peterka and Blazejczyk-Okolewska, 2004), two degrees of freedom (e.g., Awrejcewicz and Lamarque, 2003; Babitskii, 1978; Bapat, 1998; Blazejczyk-Okolewska and Kapitaniak, 1996; Cempel, 1970; Peterka, 1971, 1981; Peterka and Blazejczyk-Okolewska, 2004), three degrees of freedom (e.g., Cempel, 1970), etc.;

(B) number of limiting stops (fenders) – with one-sided limiting stops (e.g., Cempel, 1970; Peterka, 1971, 1981) or two-sided limiting stops (e.g., Blazejczyk-Okolewska et al., 1999; Blazejczyk-Okolewska and Kapitaniak, 1996; Cempel, 1970; Peterka, 1971, 1981; Peterka and Blazejczyk-Okolewska, 2004);

(C) way the limiting stops displace (e.g., Peterka, 1971; Peterka and Blazejczyk-Okolewska, 2004) or do not displace (e.g., Peterka, 1981);

(D) designs of the supporting structure – systems in which the supporting structures of subsystems depend on one another (e.g., Blazejczyk-Okolewska et al., 1999; Blazejczyk-Okolewska and Kapitaniak, 1996; Cempel, 1970) and systems with the subsystems that have independent supporting structures (e.g., Blazejczyk-Okolewska et al., 2001; Cempel, 1970);

(E) type of forces that occur in the system – elasticity forces (e.g., Bajkowski, 1996; Bapat, 1998; Blazejczyk-Okolewska et al., 1999) and energy dissipation forces as, for instance, viscous damping forces (e.g., Blazejczyk-Okolewska et al., 1999; Peterka, 1971, 1981; Peterka and Blazejczyk-Okolewska, 2004) or friction forces (e.g., Blazejczyk-Okolewska et al., 1999; Blazejczyk-Okolewska and Kapitaniak, 1996; Chin et al., 1994; Hinrichs et al., 1997; Peterka, 1981);

(F) number of excitations applied – to one body (e.g., Awrejcewicz and Lamarque, 2003; Babitskii, 1978; Bajkowski, 1996; Bapat, 1998; Blazejczyk-Okolewska et al., 1999; Blazejczyk-Okolewska and Kapitaniak, 1996; Cempel, 1970; Chin et al., 1994; Fu and Paul, 1969; Goyda and The, 1980; Hinrichs et al., 1997; Kaharaman and Singh, 1990; Lin and Bapat, 1993; Mashri and Caughey, 1966; Natsiavas, 1993; Nguyen et al., 1987; Nigm and Shabana, 1983; Nordmark, 1991; Peterka, 1971, 1981; Peterka and Blazejczyk-Okolewska, 2004; Senator, 1970; Shaw and Holmes, 1983; Tung and Shaw, 1988) or to two or more bodies (e.g., Blazejczyk-Okolewska et al., 2001; Luo and Xie, 2002);

(G) kind of excitation – kinematic (e.g., Lin and Bapat, 1993) or dynamic (e.g., Peterka, 1971, 1981);

(H) characteristics of the forces analysed in the system – elasticity forces: linear (e.g., Peterka and Blazejczyk-Okolewska, 2004) and nonlinear (e.g., Blazejczyk-Okolewska et al., 2001; Shaw and Holmes, 1983), damping forces: linear (e.g., Peterka, 1981; Peterka and Blazejczyk-Okolewska, 2004) and nonlinear (e.g., Mashri, 1966), friction forces: linear (e.g., Blazejczyk-Okolewska and Kapitaniak, 1996; Peterka, 1981) and nonlinear (e.g., Blazejczyk-Okolewska and Kapitaniak, 1996);

(I) kind of limiting stops – rigid limiting stops (e.g., Blazejczyk-Okolewska and Kapitaniak, 1996; Chin et al., 1994; Nordmark, 1991; Peterka and Blazejczyk-Okolewska, 2004), soft limiting stops (e.g., Kaharaman and Singh, 1990; Lin and Bapat, 1993; Natsiavas, 1993; Shaw and Holmes, 1983).

The division of mechanical systems with impacts with respect to the kind of limiting stops (item (I)) has been made on the basis of possible ways of impact modelling. In both cases, the coefficient of stiffness k is the decisive one. A simplified way of impact modelling consists in the assumption that the impact duration is infinitely short and the coefficient of restitution that represents energy dissipation has a constant value. Then, the way the impact occurs becomes closer to an impact on a rigid limiting stop ($k \to \infty$). This way of impact modelling is usually not sufficient, as the coefficient of restitution depends on the impact velocity and the impact duration is not infinitely short. Thus, some new tendencies of building models that allow for a more correct description of the impact process, which has a finite time of duration and becomes closer to an impact on a soft limiting stop, have appeared. In this case, we are able to select the way limiting stops are modelled (linear and nonlinear structures, for instance elastic, elastic-damping or triple combinations). In the literature, such systems are often referred to as piecewise linear oscillators, as in Show and Holmes (1983) or impact oscillators with clearance, as in Lin and Bapat (1993).

A comparison of vibro-impact systems has been made above and some their characteristic features have been pointed out. Owing to the fact that the majority of the systems analysed in the references quoted is characterised by a few features described here at the same time, a proposition has arisen that it is possible to define *types of mechanical systems with impacts*, in other words, models, to which a certain series of forms of defined structures of component elements (referred to as subsystems further on) and of defined characteristic features will correspond. Analysing the above-mentioned, one can state that at least two rigid bodies (rigid elements that cannot be divided into any other elements, such that one of them can be the frame – then the system becomes a system with one degree of freedom, as, e.g., Cempel (1970), Chin et al. (1994)), which dependent on each other functionally and such that an excitation force which causes a change in the state of the whole system acts on one of them at least, constitute a type of the system. These elements can be connected with each other in a different way, which allows for reconstructing reological properties of the system under consideration. The term reological properties refers here to the relationships of internal forces acting between individual masses of the model that are caused by their displacements with respect to one another and by time.

In the literature devoted to the subject scope considered here, the notion of "types of mechanical systems with impacts" have appeared with reference to vibration dampers. Several types of impact dampers were investigated by Peterka (1971, 1981). Paper I (Peterka, 1971) explains theoretically the general properties of the fundamental periodic motion of three essential types of impact dampers (Fig. 1(a)–(c)). Its main contribution lies in the general determination of the possible existence of two different solutions of the periodic motion and in the derivation of explicit equations for the stability boundaries. The properties of the motion of impact dampers with two-sided impacts and elastic coupling of masses are described in detail in paper II (Peterka, 1971). In paper III (Peterka, 1971), the author indicated two possible ways of application of dampers for which the optimum values of parameters were determined and the amplitude characteristics of damped mass motion were established. The influence of the dry friction force acting on the relative motion of masses of the system described in Peterka (1971) was shown in Blazejczyk-Okolewska et al. (1999). Monograph (Peterka, 1981) includes a collection of schemes of various impact vibration dampers (Fig. 1(b)–(e)) and schemes of different types of systems with impacts (Fig. 2). Fig. 2 is a reproduction of the original figure from the paper by Peterka (1981) and it shows systems that can be models of structural elements in the devices for vibro-impact machining, for ramming moulding mixes, concrete, etc.

While discussing the works devoted to types of mechanical systems with impacts, one should not forget about the study by Cempel (1970), who defined the impact force of the distributional nature and obtained a simple way of the generation of motion equations of vibro-impact systems. He obtained solutions to these equations using the operational calculus, and for systems with many degrees of freedom – introducing additionally normal co-ordinates. In this way, he solved the problem of vibrations

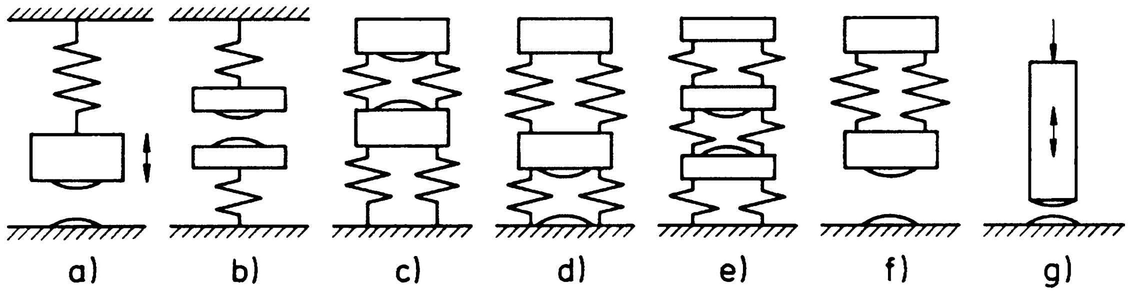


Fig. 2. Schemes of different types of structures in the devices for vibro-impact machining (Peterka, 1981).

for – as he has written – a few most important types of vibro-impact systems. For systems with one degree of freedom, Cempel differentiates between systems with one-sided and two-sided impacts (Fig. 2(a) and Fig. 1(g), correspondingly). Systems with two and many degrees of freedom are divided by him into systems with: impacts on the barrier (Fig. 1(f)), internal impacts in the system (Fig. 1(b)), impacts of two independent systems (Fig. 1(e)), and impacts of the main system on the semi-defined one (Fig. 1(c)).

During a few years of the investigations on systems with impacts that have been conducted by the authors, the following question has arisen: how many types of systems with impacts exist in general and what these types are. It has been established that systems with impacts have not been classified yet and actually there is no publication in which their number is stated clearly and which presents all types of such systems. The basic reason that has made the authors start investigating this issue is the fact that the described scientific problem exists, concerns a wide range of technical devices and has not been solved so far.

Thus, the scientific objective of this study is to present the way in which subsequent types of mechanical systems with impacts arise and to develop principles of their classification.

The significance of the problem under consideration is manifested by a great number of publications devoted to motion of systems with impacts. In these publications, many applications can be met, for instance: physical models of buildings that are used to predict effects of earthquakes (Natsiavas, 1993; Nigm and Shabana, 1983), pile-drivers for piles or pipes in oil mining, rammers for moulding mixes, crushers, riveting presses, hammer drills (Babitskii, 1978; Bajkowski, 1996; Fu and Paul, 1969; Kobrinskii and Kobrinskii, 1973; Senator, 1970), vibration dampers (especially in devices working under high temperatures and in railway engineering) (Bajkowski, 1996; Bapat, 1998; Mashri and Caughey, 1966; Nguyen et al., 1987; Peterka, 1971; Peterka and Blazejczyk-Okolewska, 2004), low-loaded toothed and cam gears (Kaharaman and Singh, 1990; Lin and Bapat, 1993; Natsiavas, 1993; Nguyen et al., 1987), vibrating conveyors, bar screens, gun lock mechanisms, electric automatic cut-outs (Nguyen et al., 1987), printing heads in needle printers (Babitskii, 1978; Bapat, 1998; Fu and Paul, 1969; Kobrinskii and Kobrinskii, 1973; Senator, 1970; Tung and Shaw, 1988), heat exchangers (Blazejczyk-Okolewska and Czolczynski, 1998; Goyda and The, 1980; Lin and Bapat, 1993).

2. Fundamental assumptions

The basic classification principles of types of mechanical systems with impacts are strictly connected with the notion of *degrees of freedom*, that is to say, the n number of independent co-ordinates that define the system configuration.

One of the first assumptions is as follows: the models of systems under consideration are rigid bodies connected by means of, for instance, springs, which can move along the straight line without a possibility of rotations. A classical example of the vibrating mechanical system with one degree of freedom ($n = 1$) is a body with the mass m suspended on a spring with the stiffness k in such a way that it can move along one straight line only. Then, the quantity x defines explicitly its position with respect to the static equilibrium position (Fig. 3(a)). The most general undamped system with two degrees of freedom can be depicted as in Fig. 3(b). It consists of two bodies with the masses m_1 and m_2, respectively, suspended on the springs k_1 and k_2 and connected by the coupling spring k_{12}. Assuming that the bodies can move along the vertical line only (the basic assumption) and that both the masses can move independently, this system has two degrees of freedom ($n = 2$). Giving the

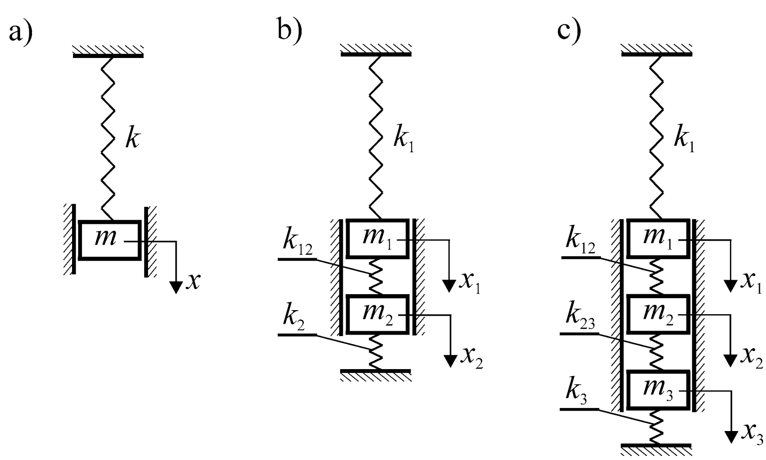


Fig. 3. Schemes of mechanical systems: (a) basic spring system for $n = 1$, (b) basic spring system for $n = 2$, (c) system with $n = 3$ that is not the basic spring system.

values x_1 and x_2, we define explicitly the position of the whole system. The body with the mass m_1 defined by the position x_1 will be referred to as the first subsystem, whereas the body with the mass m_2 described by the position x_2 will be called the second subsystem. In the light of the above-mentioned, the number of subsystems is equal to the number of degrees of freedom. A system composed of three subsystems is a system with three degrees of freedom, in which the quantities $x_1,\ x_2,\ x_3$ define explicitly the system position (Fig. 3(c)). In general, it can be said that a system has n degrees of freedom (i.e., n subsystems) if its position is described by n quantities. Fig. 3 shows basic schemes of systems with one (Fig. 3(a)), two (Fig. 3(b)) and three degrees of freedom (Fig. 3(c)), in which elastic elements are depicted as springs that play the role of a connecting structure between the subsystems ($k_{12},\ k_{23}$), that is to say, between the subsystems and the frame (k, k_1, k_2, k_3).

The next assumption consists in neglecting the mass of elastic elements, which causes that their dynamic characteristics coincides with the static characteristics in the range of positioning forces. In real mechanical systems, the forces that dissipate energy always occur apart from these forces. They can be damping forces (the symbol denoting a damper occurs in the physical model) or other forces, e.g., friction forces (the symbol representing friction is used then in the physical model), which are not represented in Fig. 3.

In the below-described classification of types of mechanical systems with impacts, proposed by the authors, the forces dissipating energy (except impact forces of course) have been neglected in order to simplify the procedure in the initial stage of the analysis. The rules of this classification are as follows:

(1) We determine the number n, i.e., the number of degrees of freedom of the mechanical system. We *do not change* this number while comparing the selected masses to zero or the selected stiffnesses or masses to the value equal to infinity. On the basis of this number, we decide on the number of subsystems (the number of degrees of freedom equals to the number of subsystems).

(2) We build a system with the specified number of degrees of freedom, on the basis of two rules, namely:

(a) Each subsystem is connected with another one by a spring. Each subsystem is connected with the frame also by a spring. In this way, the *basic spring system* with the number of springs s is formed:

$$s = n + \frac{n(n-1)}{2}. \tag{1}$$

This formula can be justified in a simple way. The basic spring system with one degree of freedom can look as the one in Fig. 3(a) ($n = 1,\ s = 1$). The basic spring system with two degrees of freedom can look as the one in Fig. 3(b) ($n = 2,\ s = 3$). Fig. 3(c), despite the fact that it represents a scheme of the system with three degrees of freedom, *does not show* – according to the author's assumptions – the basic system with three degrees of freedom. On this scheme, two spring connections are lacking – the spring k_2 connecting the body of the mass m_2 with the frame and the spring k_{13} connecting the body of the mass m_1 with the body of the mass m_3. The basic spring system with three degrees of freedom is shown in Fig. 4 ($n = 3,\ s = 6$).

(b) Each subsystem impacts on any other subsystem and the frame at both possible senses of the relative velocity. Thus, the *basic impact system* with the number of fenders z is formed:

$$z = n(n+1). \tag{2}$$

This formula can be justified in an easy way. The basic impact system with one degree of freedom ($n = 1$) can look as the one in Fig. 5(a), and the number of its fenders is equal to two ($z = 2$). The symbol "$\top$ " in the figure denotes that the upper fender $z_{1\mathrm{g}}$ occurs, whereas the symbol "$\perp$" means that the lower fender $z_{1\mathrm{d}}$ occurs, and both the fenders can impact on the frame. The basic impact system with two degrees of freedom ($n = 2$) can look as the one in Fig. 5(b), and the number of its fenders equals six ($z = 6$). Each subsystem has two fenders impacting on the frame – for the subsystem of the mass m_1, they are the fenders $z_{1\mathrm{g}}$ and $z_{1\mathrm{d}}$, whereas for the subsystem of the mass m_2 – the fenders denoted by $z_{2\mathrm{g}}$ and $z_{2\mathrm{d}}$ (the upper and lower fender, correspondingly), and the two fenders $z_{12\mathrm{g}}$ and $z_{12\mathrm{d}}$ (the upper inner fender and the lower inner fender, respectively) that

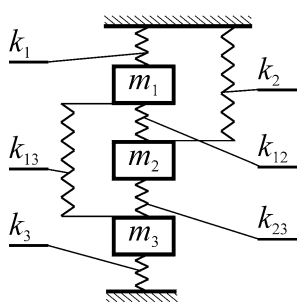


Fig. 4. Basic spring system for $n = 3$.

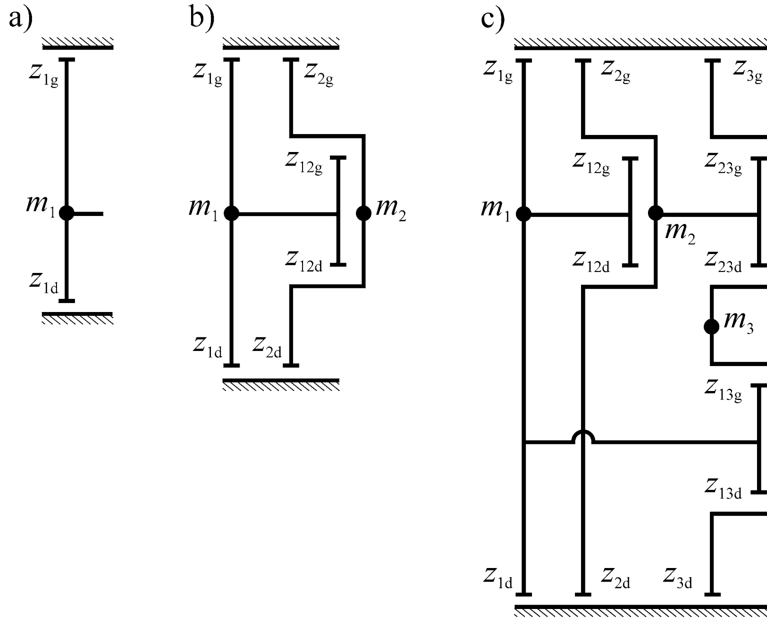


Fig. 5. Basic impact systems: (a) $n = 1$, (b) $n = 3$, (c) $n = 3$.

enable impacts between the subsystems. The basic impact system with three degrees of freedom $(n = 3)$ is shown in Fig. 5(c), and the number of its fenders is equal to twelve $(z = 12)$. Each subsystem has two fenders that impact on the frame – for the subsystem of the mass m_1, they are the fenders denoted by z_{1g} and z_{1d}, for the subsystem of the mass m_2 – z_{2g} and z_{2d}, for the subsystem of the mass m_3 – z_{3g} and z_{3d}, and six inner fenders that enable impacts between the subsystems. They are referred to as follows: z_{12g} and z_{12d} (the inner fenders between subsystems 1 and 2), z_{13g} and z_{13d} (the inner fenders between subsystems 1 and 3), z_{23g} and z_{23d} (the inner fenders between subsystems 2 and 3). Let us notice that the symbol of the fender used on drawings does not impose the way of modelling. Thus, this phenomenon can be modelled depending on the physical circumstances that are to be considered and solved.

There is a relationship between the number of fenders z and the number of springs s such that:

$$z = 2s. \tag{3}$$

Combining principles (a) and (b), we form one *basic spring–impact system*, in which every subsystem is connected with any other subsystem (each subsystem is connected with the frame as well) and it impacts on any other subsystem (every subsystem impacts on the frame as well). Fig. 6 shows basic spring–impact systems for the system with one degree of freedom (Fig. 6(a)), for the system with two degrees of freedom (Fig. 6(b)), and for the system with three degrees of freedom (Fig. 6(c)).

Let us notice that if we remove even one spring from the basic system (or even one fender), we obtain another system, which is a particular case of the basic spring–impact system.

In order to determine the number of particular cases of the spring–impact system, that is to say, of possible combinations of arrangements of springs and fenders, we should form particular cases of the basic spring system, eliminating one spring, two springs, etc., from the basic spring system (i.e., the one in which there is a certain number of springs, but there are no fenders at all – for systems $n = 1$, $n = 2$ and $n = 3$ – Figs. 3(a), (b) and Fig. 4, correspondingly). The number of particular cases of spring combinations (with various configurations of springs) is determined by the following formula:

$$i_s = 2^s. \tag{4}$$

On the other hand, particular cases of the basic impact system should be formed from the basic impact system (i.e., the one in which there is a certain number of fenders, but there are no springs – for systems $n = 1$, $n = 2$ and $n = 3$, – Figs. 5(a)–(c), respectively), eliminating one fender, two fenders, etc. The number of particular cases of impact combinations (with different configurations of fenders) is determined by the following formula:

$$i_z = 2^z. \tag{5}$$

In the case of a one-degree-of-freedom system $i_s = 2$, $i_z = 4$. A thorough analysis for $n = 1$ is included in Section 4. In the case of a system with two degrees of freedom $i_s = 8$, $i_z = 64$. A detailed analysis for $n = 2$ is presented in Section 3.

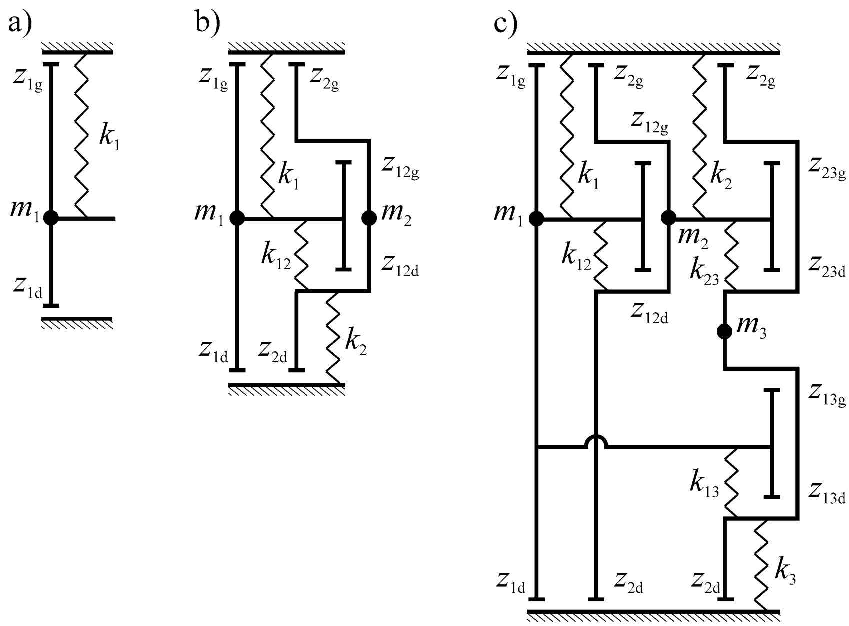


Fig. 6. Basic spring–impact systems: (a) $n = 1$, (b) $n = 3$, (c) $n = 3$.

Table 1
Exemplary way of the formation of spring–impact apt and inapt combinations for $n = 2$

	$i_{a-s} = 4$ – examples of spring apt combination configurations	$i_{i-s} = 4$ – examples of spring inapt combination configurations
$i_{a-z} = 48$ – examples of impact apt combination configurations	$i_{a-sz} = i_{a-s} \times i_{a-z} = 4 \times 48 = 192$ – examples of spring–impact apt combination configurations	$i_{a-sz} = i_{i-s} \times i_{a-z} = 4 \times 48 = 192$ – examples of spring–impact apt combination configurations
$i_{i-z} = 16$ – examples of impact inapt combination configurations	$i_{a-sz} = i_{a-s} \times i_{i-z} = 4 \times 16 = 64$ – examples of spring–impact apt combination configurations	$i_{i-sz} = i_{i-s} \times i_{i-z} = 4 \times 16 = 64$ – examples of spring–impact inapt combination configurations

We obtain all possible particular cases of *the basic spring–impact system* matching each case of the spring configuration of *the basic spring system* with each case of the fender configuration of *the basic impact system*.

This phase is called phase I (multiplication phase), and the number of all possible combinations of arrangements of springs and fenders (i.e., particular cases of the basic spring–impact system) is determined by the following formula:

$$i_{sz} = 2^s \cdot 2^z = 2^{s+z}. \tag{6}$$

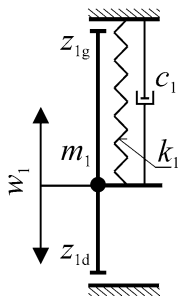
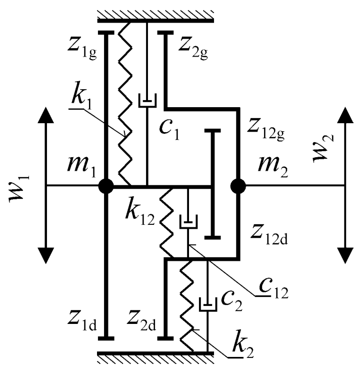
In the case of a system with one degree of freedom, the number $i_{sz} = 8$, for a two-degree-of-freedom system – $i_{sz} = 512$.

Formula (6) is the formula in which only the presence of springs and fenders is taken into consideration, and thus it does not account for the presence of other elements that connect the subsystems, for instance dampers or external excitation. This piece of information is very important, bearing in mind the fact that in the case when also damping connections are taken into account, then the number of possible combinations alters significantly (for instance, for a system with two degrees of freedom with the maximum number of springs, fenders and dampers, from 512 to 4096 – see Table 1). Although there is not any real system without damping, it has been resolved to neglect damping in the initial phase of the development of classification principles. It will simplify the calculations and facilitate the analysis of the generation method of types of systems with impacts. In the initial analysis, the effect of the external excitation has been neglected as well, as it also changes the number of possible combinations of particular cases (see Table 2).

Phase II (elimination of inapt combinations) comprises three subphases, namely:

(a) Subphase I: Elimination of these particular cases, in which the basic spring–impact system has been divided into two or more independent systems that are connected neither by a spring nor by impact. As far as a two-degree-of-freedom system

Table 2
Configuration cases for $n = 1$ and $n = 2$

	Number of spring–impact configuration cases $i_{sz} = 2^{s+z}$	Number of spring–impact–damper configuration cases $i_{szd} = 2^{s+z+d}$	Number of spring–impact–damper–excitation configuration cases $i_{szde} = 2^{s+z+d+e}$
System 1. System with one degree of freedom with the complete configuration of connections 	$i_{sz} = 8$ $1/s \to k_1/z \to z_{1g} \to z_{1d}/d \to 0/e \to 0$	$i_{szd} = 16$ $1/s \to k_1/z \to z_{1g} \to z_{1d}/d \to c_1/e \to 0$	$i_{szde} = 32$ formula (7)
System 2. System with two degrees of freedom with the complete configuration of connections 	$i_{sz} = 512$ including: – apt combinations $i_{a-sz} = 448$, – inapt combinations $i_{i-sz} = 64$ $2/s \to k_1 \to k_{12} \to k_2/z \to z_{1g} \to z_{2g} \to z_{12g} \to z_{12d} \to z_{1d} \to z_{2d}/d \to 0/e \to 0$	$i_{szd} = 4096$ including: – apt combinations $i_{a-szd} = 3840$, – inapt combinations $i_{i-szd} = 256$ $2/s \to k_1 \to k_{12} \to k_2/z \to z_{1g} \to z_{2g} \to z_{12g} \to z_{12d} \to z_{1d} \to z_{2d}/d \to c_1 \to c_{12} \to c_2/e \to 0$	$i_{szde} = 16384$ including: – apt combinations $i_{a-szde} = 15360$, – inapt combinations $i_{i-szde} = 1024$ formula (8)

is concerned, the system presented in Fig. 7(a), which does not have either the spring k_{12} or the inner fenders z_{12g} and z_{12d} is an example and therefore it should be eliminated in the spring–impact classification.

(b) Subphase II: Elimination of the particular cases that are identical because of:
- symmetry of the systems (see, e.g., Fig. 7 (b) and (c)),
- symmetry of the systems after changes in the numbers referring to subsystems (the last subsystem is the first one, the system one but last is the second one, and the system two but last is the third one, etc., see Fig. 7 (d) and (e)).

The system depicted in Fig. 7(b) is capable of performing identical motions as the system in Fig. 7(c) (assuming the reverse sense of the frame of reference), thus one of them should be eliminated. A slightly different situation is presented in Fig. 7 (d) and (e). The body of the mass m_1 from Fig. 7(d) can perform exactly the same motions as the motions of the body with the mass m_2 from Fig. 7(e) (assuming the reverse sense of the frame of reference) and, vice versa, the body of the mass m_2 from Fig. 7(d) performs identical motions as the motions of the body with the mass m_1 from Fig. 7(e). From the viewpoint of the theory of spring–impact classification presented above, one of the systems should be eliminated.

It should be added that for a two-degree-of-freedom system, such particular cases, in which the basic spring–impact system is divided into two independent systems and cases identical because of the symmetry and the symmetry after changes in the numbers referring to subsystems, are significantly more numerous. Therefore, the analysis of subphase I and subphase II will be presented in detail in the further considerations concerning the classification.

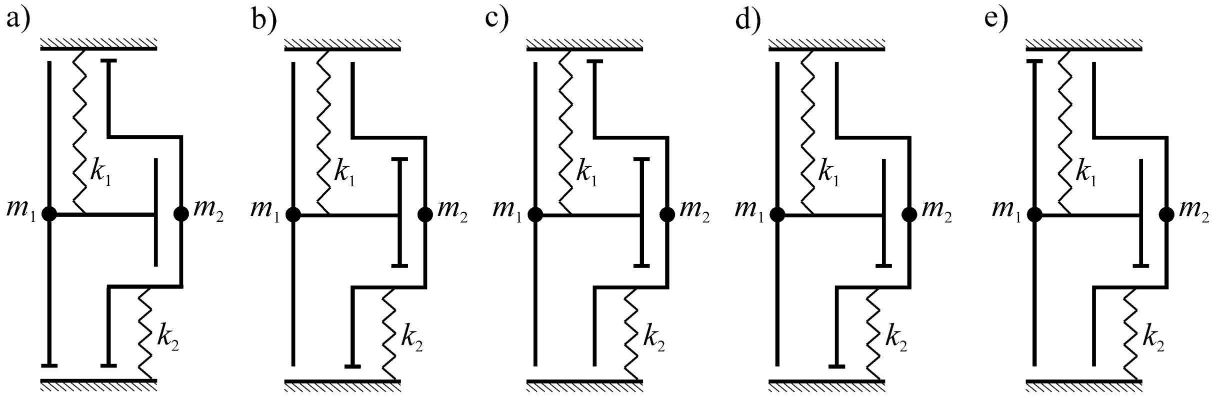


Fig. 7. Examples of the spring–impact combinations for $n = 2$; (a) inapt combination in which the basic spring–impact system is divided into two subsystems, (b) and (c) cases identical due to the symmetry of the systems, (d) and (e) cases identical due to the symmetry of the systems after changes in the numbers referring to individual subsystems.

(a) Subphase III: Elimination of these particular cases that differ because they have a given fender or do not have it in the situation when the fender does not impact on anything at all (*passive fender*). The elimination criteria in this subphase follow from the relationships between the dimensions of the basic spring–impact system (i.e., the geometrical dimensions are meant here), which are to be described further in the considerations concerning the classification.

It has been stated that the inaptness of combinations of whole groups of variants of springs and fenders can be predicted on the basis of their specific properties. For instance, a system with two degrees of freedom that arises from a combination of a spring variant with a whole group of variants of fenders is characterised by the fact that it does not admit (predict) possibilities of mutual impacts due to a lack of internal fenders and a lack of an inner spring connection.

Now, let us introduce the notion of the *zone of aptness*.

In each combination originated from the basic spring system (Fig. 8) and in each combination that arises from the basic impact system (Figs. 9–12), there are some features that decide about the aptness. These features will be referred to as *the zone of aptness.* The zone of aptness plays a role of the informant between the subsystems and if it exists in a given combination, each body gets information (learns) about the existence of other bodies thanks to it. On the schemes representing spring combinations, it is *the zone of spring aptness,* whereas on the schemes showing impact combinations – *the zone of impact aptness*.

The above-mentioned rules will be discussed in detail for a system with two degrees of freedom and a system with one degree of freedom.

3. Classification principles of systems with two degrees of freedom

As the first one, a system with two degrees of freedom, i.e., $n = 2$, has been considered. This is the system, on whose example the way of reasoning and the classification of possible variants, can be shown in the easiest way.

The maximum number of springs (spring connections), according to formula (1), equals $s = 3$ (k_1 – spring connecting the mass m_1 with the frame, k_{12} – spring connecting the mass m_1 with the mass m_2, k_2 – spring connecting the mass m_2 with the frame). All possible springs and their symbols are represented in Fig. 6(b) or in System 2, Table 2. The number of possible combinations of systems from the basic spring system is determined from formula (4), which yields the result $i_s = 8$. In Fig. 8, all possible combinations of springs for a two-degree-of-freedom system have been depicted.

The zone of spring aptness lies between mass 1 and 2, and this zone is formed by one spring k_{12} only, which decides whether the basic spring system will be divided into two subsystems or not (in this case, only two subsystems). In the case of another system, a division into two or more subsystems can occur of course and the presence of a higher number of springs can decide about this division. If we reduce the total number of springs (three springs) by the number of springs from the zone of spring aptness (one spring), then we will obtain the number of springs equal to two, which yields the number of possible combinations beyond the zone of spring aptness equal to $2^2 = 4$. This number multiplied by the number of combinations of these spring connections that decide about the aptness (there is one such a connection), that is to say $i_{a-s} = 4 \times 1 = 4$, is the number of spring apt combinations (see Fig. 8 – combinations of springs (1)–(4)). The number of possible combinations beyond the spring aptness zone multiplied by the number of combinations of these spring connections that decide about the inaptness (there is one such a connection), that is to say $i_{i-s} = 4 \times 1 = 4$, is the number of spring inapt combinations (see Fig. 8 – combinations

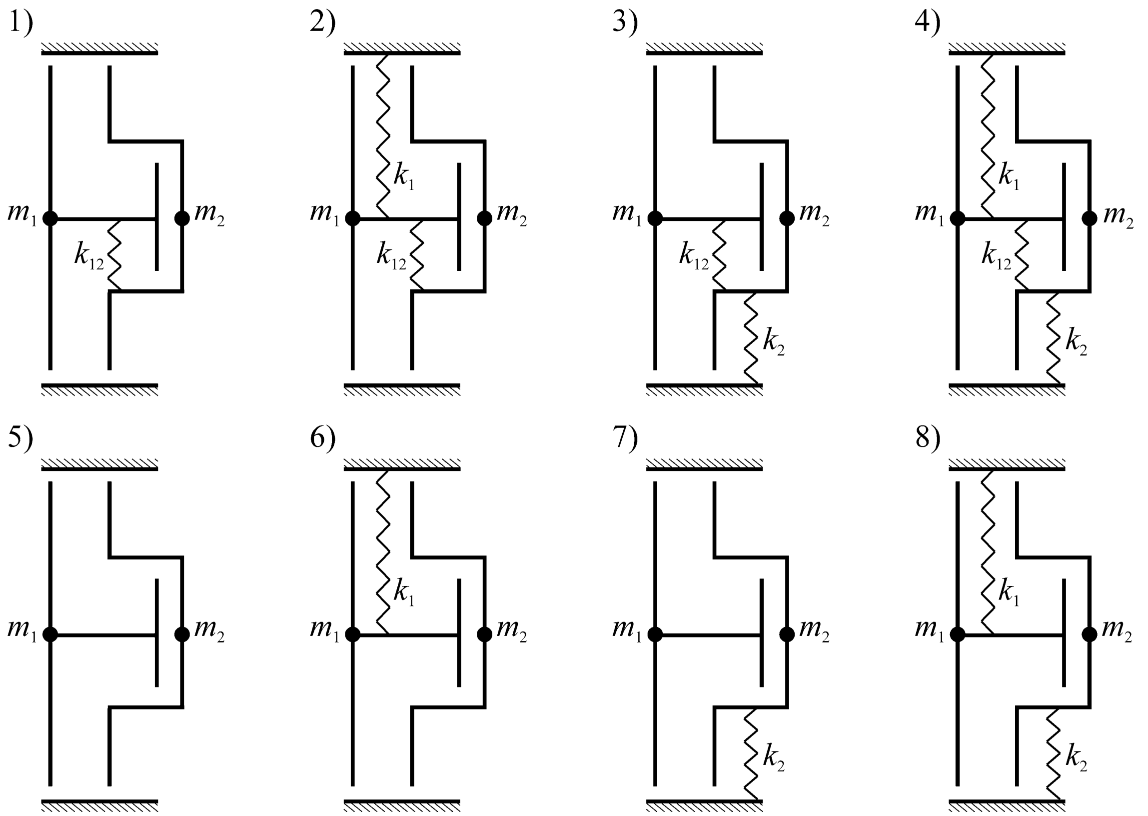


Fig. 8. Possible spring combinations that arise from the basic spring system for $n = 2$; (1)–(4) with the spring aptness zone (spring k_{12}), (5)–(8) – without the spring aptness zone.

of springs (5)–(8)). The number of spring apt combinations added to the number of spring inapt combinations gives the result equal to the previously calculated one, according to formula (4) $i_s = i_{a-s} + i_{i-s} = 4 + 4 = 8$. Fig. 8 (combinations of springs (1)–(8)) shows all possible combinations originated from the basic spring system for a system with two degrees of freedom. The crucial information is such that there will be no impacts in these systems as there are no fenders in them (there are no symbols $\top$ and $\perp$ defined before).

The same analysis has been carried out for impact connections. The maximum number of fenders (impact connections), according to formula (2), is equal to $z = 6$. These are the following connections: $z_{1\mathrm{g}}$ – upper impact connections of the mass m_1 with the frame, $z_{1\mathrm{d}}$ – lower impact connection of the mass m_1 with the frame, $z_{12\mathrm{g}}$ – upper impact connection of the mass m_1 with the mass m_2, $z_{12\mathrm{d}}$ – lower impact connection of the mass m_1 with the mass m_2, $z_{2\mathrm{g}}$ – upper impact connection of the mass m_2 with the frame, $z_{2\mathrm{d}}$ – lower impact connection of the mass m_2 with the frame. All possible fenders with the symbols denoting them are presented in Fig. 6(b) or in System 2, Table 2. The number of possible combinations of systems from the basic impact system is determined from formula (5), which yields the result $i_z = 64$. Combinations (1)–(16) in Figs. 9–12 show all possible combinations of impact connections for a two-degree-of-freedom system.

The zone of impact aptness lies between masses 1 and 2, and this zone is formed by two fenders $z_{12\mathrm{g}}$ and $z_{12\mathrm{d}}$, which decide whether the basic impact system will be divided into two subsystems or not (in this case, into two subsystems only). In the case of the system with $n > 2$, a division into two or more subsystems can occur of course and not only the fenders that connect mass 1 with mass 2 can decide about it. If we reduce the total number of impact connections (six fenders) by the number of fenders from the zone of impact aptness (two fenders), then we will receive the number of fenders equal to four, which yields the number of possible combinations beyond the impact aptness zone equal to $2^4 = 16$. This number multiplied by the number of combinations of these impact connections that decide about the aptness (there are three such connections), that is to say $i_{a-z} = 16 \times 3 = 48$, is the number of impact apt combinations. Combinations (1)–(16) in Figs. 9–11 show all impact apt combinations – in Fig. 9, the connection between the masses is made both through the upper impact connection $z_{12\mathrm{g}}$ and the lower one – $z_{12\mathrm{d}}$, in Fig. 10 – the connection between the masses occurs only through the upper impact connection $z_{12\mathrm{g}}$, and in Fig. 11 – the connection between the masses occurs only through the lower impact connection $z_{12\mathrm{d}}$. The number of possible combinations beyond the zone of impact aptness multiplied by the number of these impact connections that decide about the

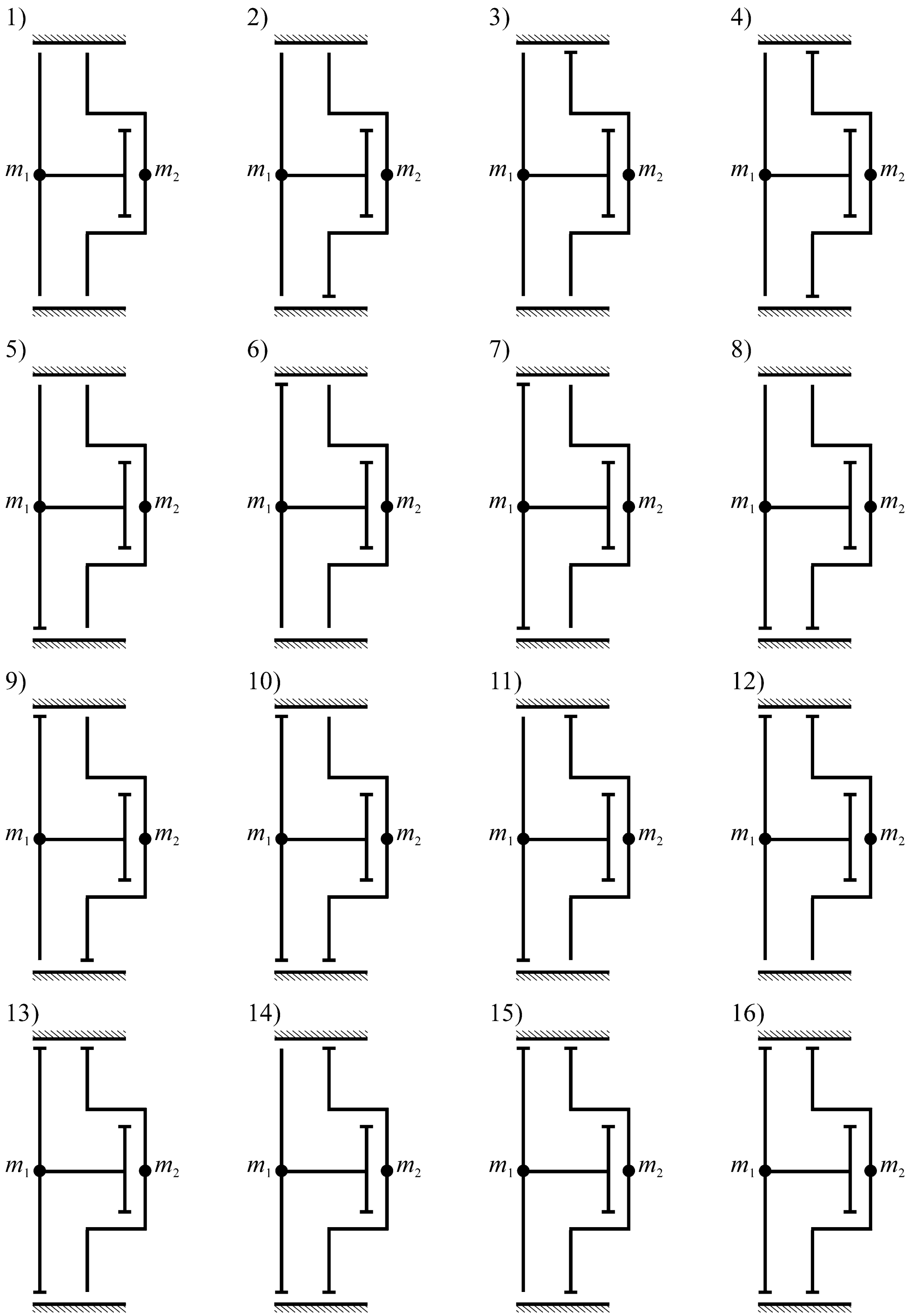


Fig. 9. Possible fender combinations that arise from the basic impact system for $n = 2$, including the impact aptness zone (two fenders z_{12g} and z_{12d}).

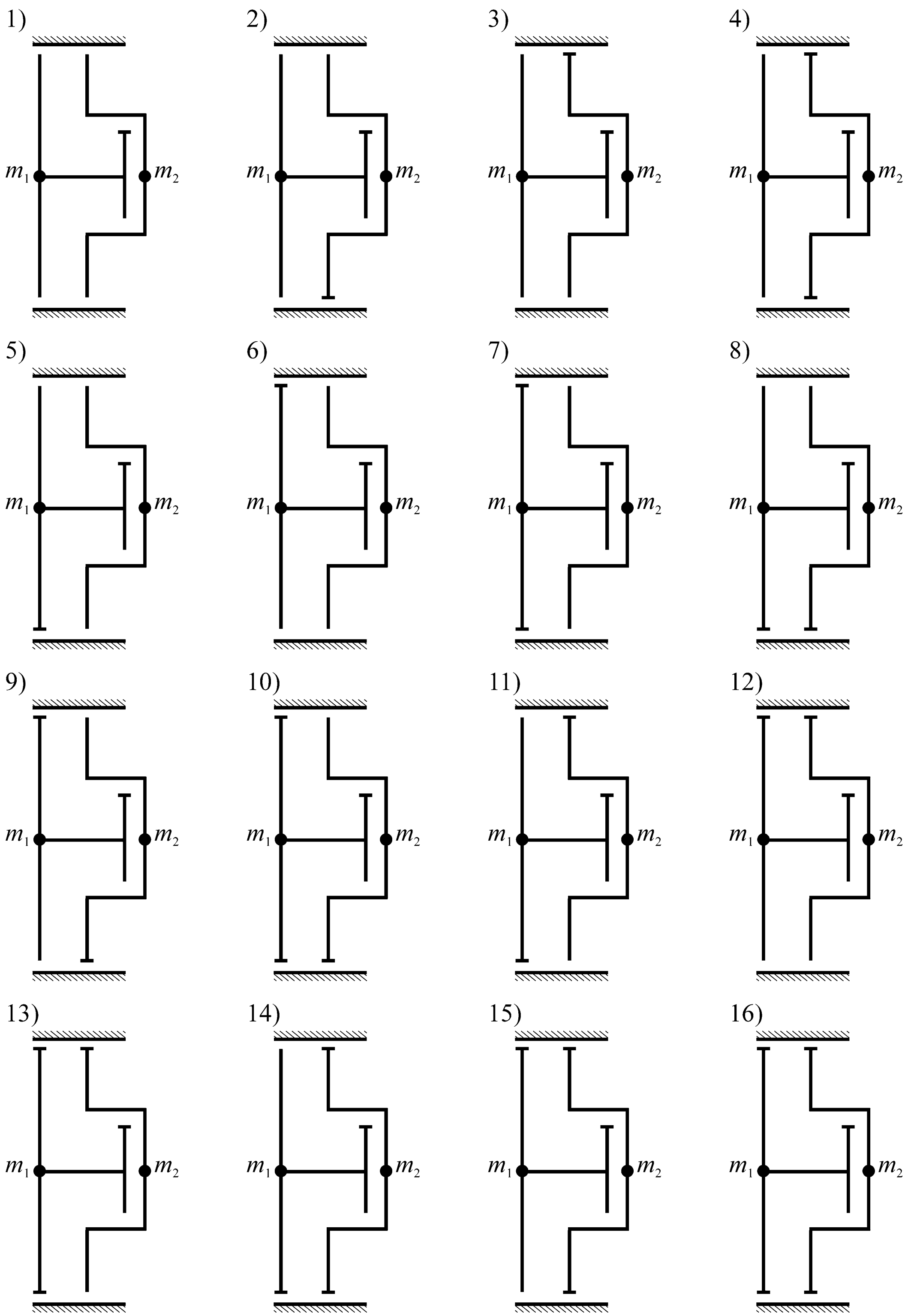


Fig. 10. Possible fender combinations that arise from the basic impact system for $n = 2$, including the impact aptness zone (fender z_{12g}).

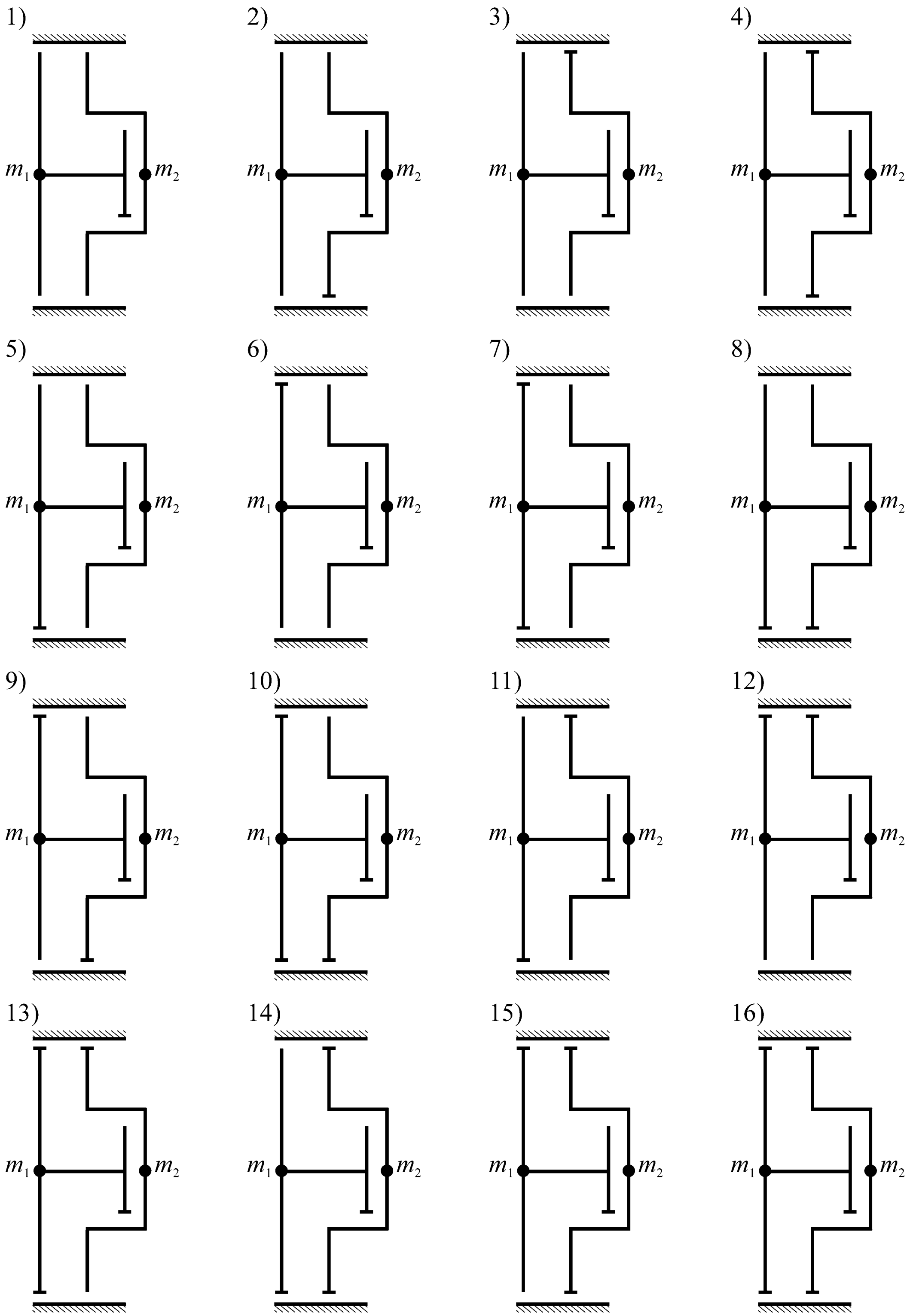


Fig. 11. Possible fender combinations that arise from the basic impact system for $n = 2$, including the impact aptness zone (fender z_{12d}).

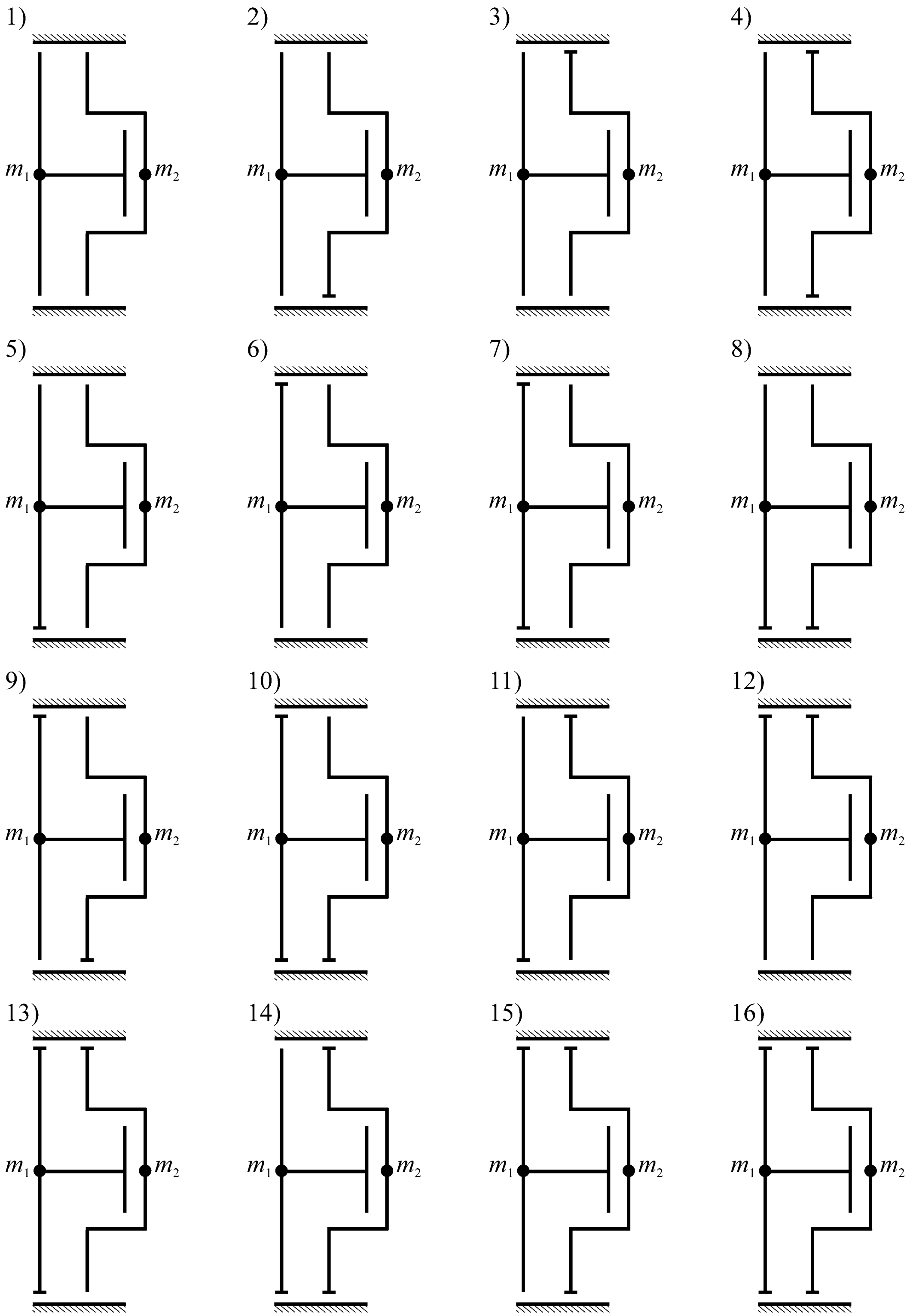


Fig. 12. Possible fender combinations that arise from the basic impact system for $n = 2$, without the impact aptness zone.

inaptness (there is only one such a connection), that is to say $i_{i-z} = 16 \times 1 = 16$, is the number of impact inapt combinations. Combinations (1)–(16) in Fig. 12 show all possible impact inapt combinations where there is neither an upper impact connection nor a lower impact connection at all. The number of impact apt combinations added to the number of impact inapt combinations yields the result equal to the result calculated before, according to formula (5), $i_z = i_{a-z} + i_{i-z} = 48 + 16 = 64$. Figs. 9–12 present all possible combinations originated from the basic impact system for a two-degree-of-freedom system.

The authors would like to draw the reader's attention to the fact that in the case of 4 spring apt combinations (Fig. 8 – combinations of springs (1)–(4)), the body of the mass m_1 will learn about the presence of another body of the mass m_2 (and vice versa) due to the occurrence of elasticity forces between these bodies (presence of the spring k_{12}). In turn, in the case of 4 spring inapt combinations (Fig. 8 – combinations of springs (5)–(8)), the body of the mass m_1 will never learn about the existence of the body of the mass m_2 (and vice versa), because the bodies will never be able to pass the information concerning their existence due to a lack of a spring connection. In the case of 48 impact apt combinations (all combinations in Figs. 9–11), the body of the mass m_1 will learn about the existence of the second body of the mass m_2 owing to the existence of impact forces between these bodies (there exist two fenders $z_{12\mathrm{g}}$ and $z_{12\mathrm{d}}$ in the case of the combinations in Fig. 9 or at least one fender in the case of the combination presented in Fig. 10 – fender $z_{12\mathrm{g}}$, or the combination in Fig. 11 – fender $z_{12\mathrm{d}}$). On the other hand, in the case of impact inapt combinations (1)–(16) in Fig. 12, the body of the mass m_1 will never learn about the body of the mass m_2 (and vice versa), because these bodies will not pass any information on their existence due to a lack of impact forces.

According to the previously assumed principle, next a matching of every particular case of the spring configuration with every particular case of the fender combinations is carried out. Because of the fact that the number of all possible cases that follow from the "each-to-each" matching is equal to 512, only one example of the matching is presented. Fig. 13 shows all spring–impact combinations that originated from the matching of the system with spring combinations (8) in Fig. 8 with the system of fender combinations (1)–(16) from Fig. 9.

The total number of spring–impact apt combinations in a system with two degrees of freedom equals $i_{a-sz} = 448$, whereas the total number of spring–impact inapt combinations is equal to $i_{i-sz} = 64$. Table 1 presents an exemplary way of the so-called "each-to-each" matching for a system with two degrees of freedom.

It is not difficult to state that as a result of the matching of a spring apt combination with an impact apt combination, a spring–impact apt combination arises. There are $i_{a-sz} = i_{a-s} \times i_{a-z} = 4 \times 48 = 192$ spring–impact apt combinations altogether and they are the matchings of all combinations of springs (1)–(4) (Fig. 8) with all combinations of fenders (1)–(16) (Figs. 9–11). As a result of the matching of a spring apt combination with an impact inapt combination, a spring–impact apt combination arises. There are $i_{a-sz} = i_{a-s} \times i_{i-z} = 4 \times 16 = 64$ spring–impact apt combinations and they are the matchings of all combinations of springs (1)–(4) (Fig. 8) with all combinations of fenders (1)–(16) (Fig. 11). As a result of the reverse matching, that is to say, of the matching of the configuration of the spring inapt combination with the configuration of an impact apt combination, also a spring–impact apt combination occurs. There are $i_{a-sz} = i_{i-s} \times i_{a-z} = 4 \times 48 = 192$ such spring–impact apt combinations altogether and they are the matchings of all combinations of springs (5)–(8) (Fig. 8) with all combinations of fenders (1)–(16) (Figs. 9–11). In turn, in the case of the matching of the configuration of a spring inapt combination with the configuration of an impact inapt combination, a spring–impact inapt combination occurs. There are $i_{i-sz} = i_{i-s} \times i_{i-z} = 4 \times 16 = 64$ such spring–impact inapt combinations altogether and they are the matchings of all combinations of springs (5)–(8) (Fig. 8) with all combinations of fenders (1)–(16) (Fig. 12).

The above-mentioned considerations referred to the presence of springs and fenders only in a mechanical system with two degrees of freedom. Fig. 6(b) shows a complete set of springs and fenders (the maximum number of springs and the maximum number of fenders) together with their denotations. It is one of the cases of a spring–impact apt configuration (the matching that resulted from the combination of springs (4) from Fig. 8 with fenders (16) from Fig. 9.

The situation becomes significantly simpler during the analysis of a system with one degree of freedom, and it becomes more difficult for a system with three and more degrees of freedom. The classification of systems with one degree of freedom is presented below, whereas the classification of systems with three and more degrees of freedom will be developed by the authors in the future.

4. Classification principles of one-degree-of-freedom systems

For a one-degree-of-freedom system, the number $n = 1$. The maximum number of springs, according to formula (1), equals $s = 1$ (k_1 – spring connecting the mass m_1 with the frame, Fig. 6(a) or System 1 in Table 2). The number of possible combinations of systems from the basic spring system is determined from formula (4), which yields the result $i_s = 2$. In Fig. 14, both possible combinations of springs for a one-degree-of-freedom system are presented.

It is difficult to speak about the spring inaptness in this case – the system can have a spring that connects it with the frame (spring combination (1) – Fig. 14) or can have no spring at all (spring combination (2) – Fig. 14), but the spring does not come

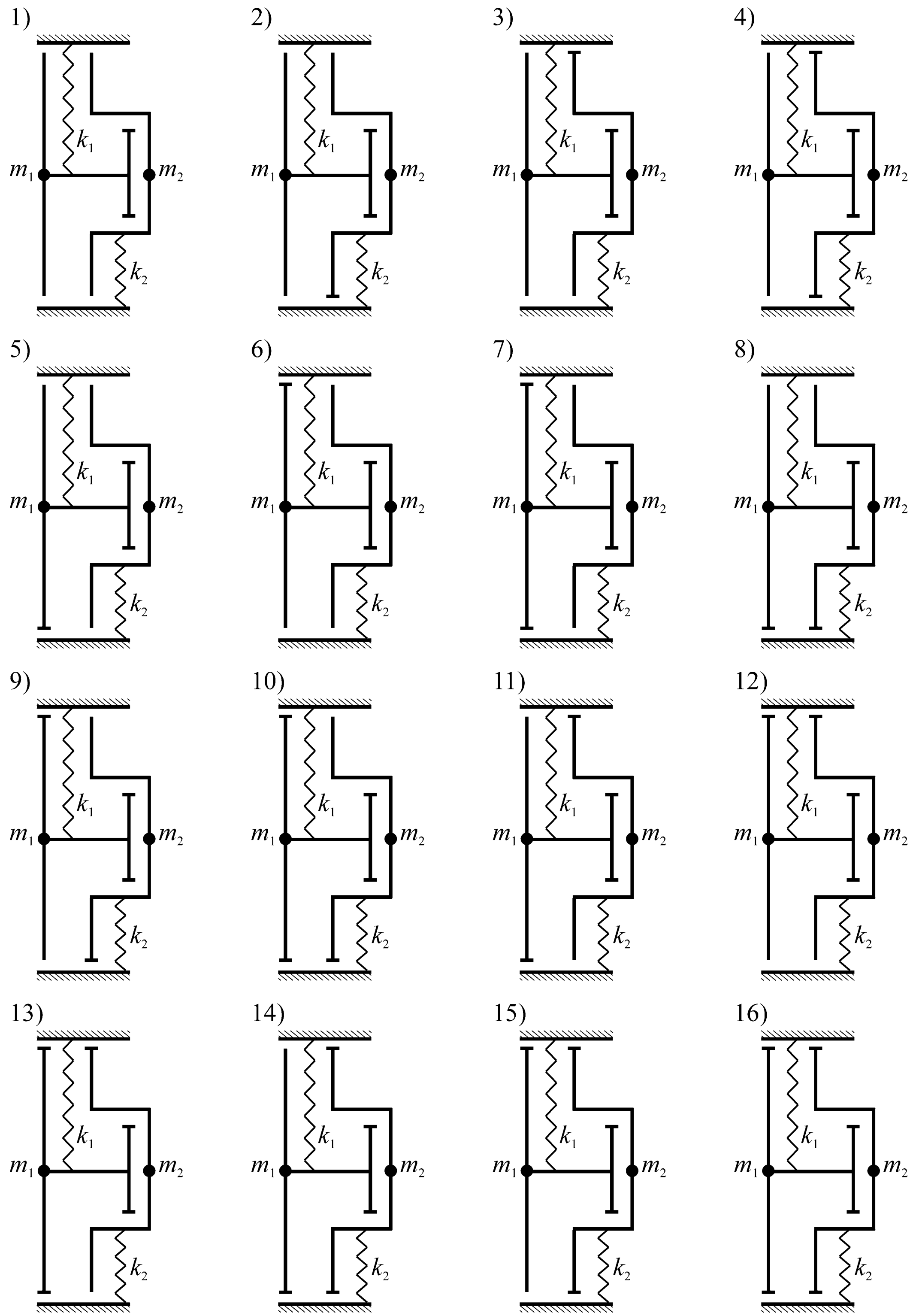


Fig. 13. Possible combinations that arise as a result of the matching of the system with spring combination (8) in Fig. 8 with the systems with fender combinations (1)–(16) in Fig. 9.

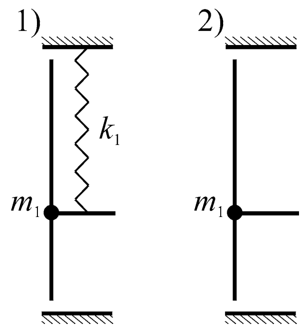


Fig. 14. Possible spring combinations that arise from the basic spring system for $n = 1$.

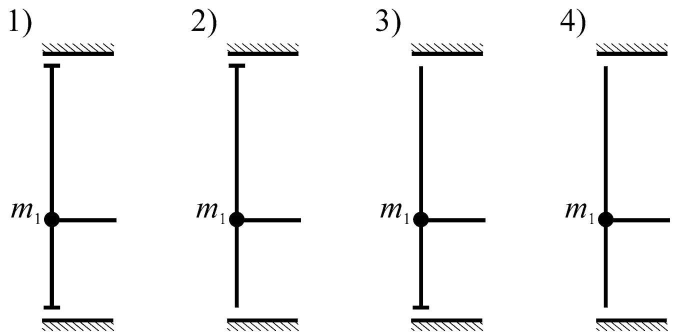


Fig. 15. Possible fender combinations that arise from the basic impact system for $n = 1$.

from the spring aptness zone. Analysing the impact connections, it has been stated that the maximum number of fenders (impact connections), according to formula (2), equals $z = 2$(z_{1g} – upper impact connection of the mass m_1 with the frame, z_{1d} – lower impact connection of the mass m_1 with the frame; both the impact connections together with their denotations are included in Fig. 6(a) or in System 1 in Table 2). The number of possible combinations of systems from the basic impact system is determined from formula (5), which gives $i_z = 4$. Fig. 15 shows all possible combinations of fenders for a one-degree-of-freedom system.

In the case of a system with one degree of freedom, it is also difficult to speak about the impact aptness: the system can have both fenders connecting it with the frame (fender combination (1) – Fig. 15) or can have just one fender at the top (fender combination (2) – Fig. 15) or at the bottom (fender combination (3) – Fig. 15) or can have no fenders at all (fender combination (4) – Fig. 15), but these fenders do not come from the impact aptness zone.

According to the previously assumed principle, a matching of both the cases of spring configuration with every case of the fender configuration takes place now. All possible cases originated from the "each-to-each" matching are presented in Fig. 16. As for a one-degree-of-freedom system, there are neither spring inapt combinations nor impact inapt combinations, then there are no spring–impact inapt combinations either. All cases of matchings for a system with one degree of freedom (Fig. 16) are the cases of spring–impact apt combinations. As there are no real systems described by the spring–impact combination (8) (Fig. 16), this case is an untypical apt combination in the classification proposed. On the other hand, if we take into account an occurrence of a damping force and external excitation in the system (for instance, a body of the mass mounted in a sleeve, on which an external force acts), then we observe a motion of the mass and it is the most correct system from the viewpoint of the theory of vibrations.

Spring–impact combinations (1)–(8) presented in Fig. 16 concern the occurrence of springs and fenders only in a one-degree-of-freedom system. Fig. 6(a) shows a complete set of springs and fenders (the maximum number of springs and the maximum number of fenders) together with their denotations. It is one of the cases of the spring–impact configuration (combination (1)) in Fig. 16.

5. Mechanical systems with damping, excitation and nonlinearity

The object of the detailed considerations included in the present paper are systems with one and two degrees of freedom. Above, a complete spring–impact classification has been presented for such systems. In the case of presence of dampers and external excitations, the number of configuration cases increases rapidly. Table 2 presents a set of the numbers of the

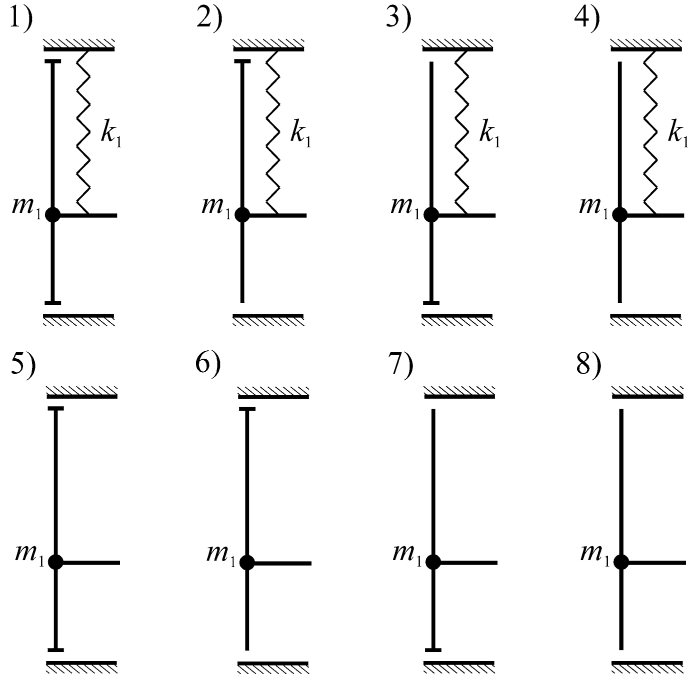


Fig. 16. Possible combinations that arise from the matching of the systems with spring combinations (1) and (2) in Fig. 14 with the systems with fender combinations (1)–(4) in Fig. 15.

configurations for a one-degree-of-freedom system and for a two-degree-of-freedom system. While analysing this table, one can state that the higher the number of degrees of freedom n, the faster the increase in the numbers: i_{sz} (number of spring–impact configurations), i_{szd} (number of spring–impact–damping configurations), i_{szde} (number of spring–impact–damper–excitation configurations).

Many practical issues can be modelled by means of a linear model with the sufficient accuracy, but a more precise description requires an application of nonlinear models. Nonlinearities of elastic and damping elements belong to typical examples in mechanical systems. Accounting for these nonlinearities instead of the linearity in the system is not followed by any changes in the number of possible configurations.

However, if we take into account (apart from damping forces) a possibility of occurrence of, for instance, a friction force, then the number of configurations grows from 32 to 64 in the case of a one-degree-of-freedom system and from 16 384 to 131 072 in the case of a two-degree-of-freedom system.

The verification how many configurations of systems exist requires the knowledge of the number of possible, quantitatively different connections (formula (2) for the impact connection and formula (1) for all the remaining connections) and then formula (6), supplemented respectively in the exponent, should be employed (Table 2).

Systems 1 and 2 from Table 2 include a complete combination of spring–impact–damper–excitation configurations. For a system with one degree of freedom, the following notation is proposed:

$$1/s \to k_1/z \to z_{1g} \to z_{1d}/d \to c_1/e \to w_1, \tag{7}$$

where:

1 – one degree of freedom,

$s \to k_1$ – spring connection by a spring of the stiffness k_1,

$z \to z_{1g} \to z_{1d}$ – impact connection by the fenders z_{1g} and z_{1d},

$d \to c_1$ – damper connection with the damping c_1,

$e \to w_1$ – action of the excitation force.

In turn, for a system with two degrees of freedom, the notation is proposed as follows:

$$2/s \to k_1 \to k_{12} \to k_2/z \to z_{1g} \to z_{2g} \to z_{12g} \to z_{12d} \to z_{1d} \to z_{2d}/d \to c_1 \to c_{12} \to c_2/e \to w_1 \to w_2, \tag{8}$$

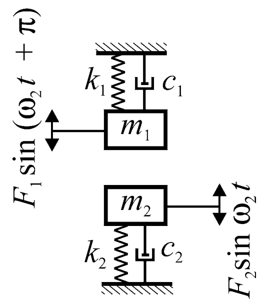
where:

2 – two degrees of freedom,

Table 3
Examples from the references and the notations of the systems considered in them, according to the classification principles

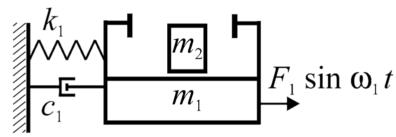
Example from the literature and notation of the system under consideration

Blazejczyk-Okolewska et al. (2001)



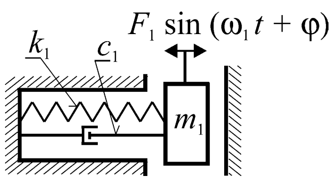
$2/s \rightarrow k_1 \rightarrow k_2/z \rightarrow z_{12\mathrm{d}}/d \rightarrow c_1 \rightarrow c_2/e \rightarrow w_1 \rightarrow w_2$

Mashri and Caughey (1966)



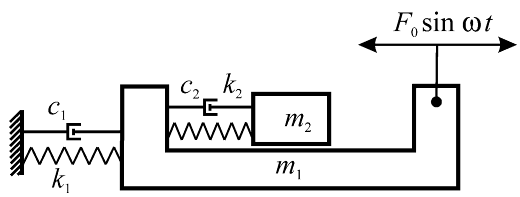
$2/s \rightarrow k_1/z \rightarrow z_{12\mathrm{g}} \rightarrow z_{12\mathrm{d}}/d \rightarrow c_1/e \rightarrow w_1$

Cempel (1970)



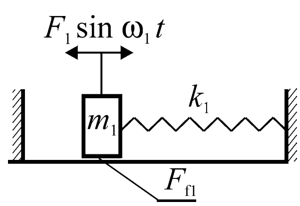
$1/s \rightarrow k_1/z \rightarrow z_{1\mathrm{g}} \rightarrow z_{1\mathrm{d}}/d \rightarrow c_1/e \rightarrow w_1$

Peterka and Blazejczyk-Okolewska (2004) in press



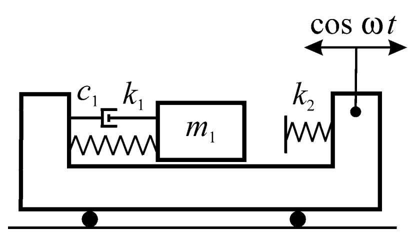
$2/s \rightarrow k_1 \rightarrow k_{12}/z \rightarrow z_{12\mathrm{g}} \rightarrow z_{12\mathrm{d}}/d \rightarrow c_1 \rightarrow c_{12}/e \rightarrow w_1$

Chin et al. (1994)



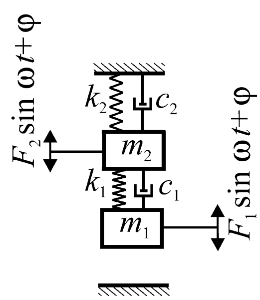
$1/s \rightarrow k_1/z \rightarrow z_{1\mathrm{d}}/f \rightarrow f_1/e \rightarrow w_1$

Shaw and Holmes (1983)



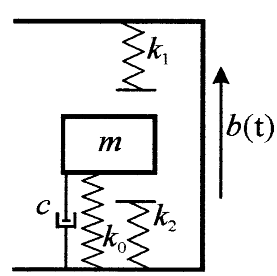
$1/s \rightarrow k_1/z \rightarrow z_{1\mathrm{d}}/d \rightarrow c_1/e \rightarrow w_1$

Luo and Xie (2002)



$2/s \rightarrow k_{12} \rightarrow k_2/z \rightarrow z_{1\mathrm{d}}/d \rightarrow c_{12} \rightarrow c_2/e \rightarrow w_1 \rightarrow w_2$

Lin and Bapat (1993)



$1/s \rightarrow k_1/z \rightarrow z_{1\mathrm{g}} \rightarrow z_{1\mathrm{d}}/d \rightarrow c_1/e \rightarrow w_1$

$s \to k_1 \to k_{12} \to k_2$ – spring connections by springs of the stiffnesses k_1 (spring connection of the subsystem of the mass m_1 with the frame), k_{12} (spring connection between the subsystems of the masses m_1 and m_2), k_2 (spring connection of the subsystem of the mass m_2 with the frame), respectively,

$z \to z_{1g} \to z_{2g} \to z_{12g} \to z_{12d} \to z_{1d} \to z_{2g}$ – impact connections by the fenders z_{1g} (impact connection of the subsystem of the mass m_1 with the upper frame), z_{2g} (impact connection of the subsystem of the mass m_2 with the upper frame), z_{12g} (upper impact connection between subsystems of the masses m_1 and m_2), z_{12d} (lower impact connection between subsystems of the masses m_1 and m_2), z_{1d} (impact connection of the subsystem of the mass m_1 with the lower frame), z_{2d} (impact connection of the subsystem of the mass m_2 with the lower frame), respectively,

$d \to c_1 \to c_{12} \to c_2$ – damper connections with the damping c_1 (damper connection of the subsystem of the mass m_1 with the frame), c_{12} (damper connection between the subsystems of the masses m_1 and m_2), c_2 (damper connection of the subsystem of the mass m_2 with the frame), respectively,

$e \to w_1 \to w_2$ – action of the excitation forces w_1 (on the subsystem of the mass m_1), w_2 (on the subsystem of the mass m_2), respectively.

In the case any connection is lacking, "zero" can be used in the notation of the system (see Table 2) or this connection can be neglected (see Table 3).

In the case of occurrence of a friction force in the physical model, the notation of the system should be supplemented with the symbol of the friction force. This symbol is to used in front of the symbol describing the excitation. The friction force can act between subsystems or between a subsystem and the frame (for $n = 1 - f \to f_1$, for $n = 2 - f \to f_1 \to f_{12} \to f_2$).

When a nonlinearity appears, the way of notation does not alter.

Table 3 includes some selected examples from the literature devoted to the subject and the notations of the systems considered there, according to the classification principles proposed. The authors of the present study would like to draw attention to the fact that in the references quoted there are descriptions of systems with various ways of modelling of the impact process. The proposed way of the notation of systems with impacts imposes certain symbolic meanings of these denotations, but it is simple and can be used in scientific studies.

6. Conclusions

The way in which subsequent types of mechanical systems with impacts with n degrees of freedom arise has been presented and their classification has been shown. The presentation of classification principles is a new compilation, according to the author's knowledge. The paper answers the question: how many types of systems with impacts exist in general and what these types are, and it leads to many conclusions, as well as shows directions of further investigations.

The subject of the detailed considerations in this paper are systems with one and two degrees of freedom. The models of systems under consideration are rigid bodies connected by means of, for instance, springs, which can move along a straight line without any rotations. For such systems, a complete spring–impact classification has been presented. In the case dampers or external excitations occur, the number of configuration cases grows rapidly. An increase in the number of configurations results also from a higher number of degrees of freedom.

The classification principles developed take into account a possibility of occurrence of, for instance, a friction force. Then, the number of possible combinations increases very sharply.

The verification how many configuration cases in the system with n degrees of freedom exist requires the knowledge of the number of possible, quantitatively different connections in the system, and then the employment of the formula that determines the number of configurations.

If a nonlinearity is taken into account in the system (for instance, the nonlinearity of elastic or damping elements) instead of a linearity, the number of possible configurations does not alter.

References quoted in the present paper include the analysis of mechanical systems with impacts and describe the systems that are particular cases of the basic spring–impact–damper–excitation system proposed by the authors of the present study. The presented types of mechanical systems constitute a set in which the way of modelling of the impact phenomenon is not the differentiating parameter. A simple way of the notation of mechanical systems with impacts, consistent with the classification principles, is provided.

The above-mentioned classification of mechanical systems is of primary importance in their designing processes. The basic conditions that have to be satisfied by the subsystems are the geometrical conditions that allow for assembling the system and the geometrical conditions that permit external and internal impacts in the system. Both the types of conditions will be developed in the future research devoted to the classification of systems with impacts.

The present study gives much information of the fundamental nature that extends the knowledge on the motion of mechanical systems with impacts. This information can be applied in computations and designing processes of the above-described structures and can serve as the basis for starting the investigations on mechanical systems with impacts whose motion is multidimensional.

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