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# Riddling bifurcation and ... interstellar journeys

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Dedicated to Mohamed El Naschie who has been on an interstellar journey in fractal space-time and returned on time to his 60th birthday

**Abstract**

We show that riddling bifurcation which is characteristic for low-dimensional attractors embedded in higher-dimensional phase space can give physical mechanism explaining interstellar journeys described in science-fiction literature.


Phase spaces of many dynamical systems describing nature are higher- or even infinite-dimensional ones. The classical examples are systems described by a ordinary differential equation. However, in many cases observable phenomena of practical importance occur on (or in the neighborhood) of low-dimensional manifold [1]. In this paper we argue that the transition from lower-dimensional attractor to the higher-dimensional one can explain the way in which interstellar journeys are described in science-fiction literature.

Consider two identical chaotic systems $x_{n+1} = f(x_n)$ and $y_{n+1} = f(y_n)$, $x, y \in \mathscr{R}$ evolving on an asymptotically stable chaotic attractor $A$ coupled as

$$\begin{aligned} x_{n+1} &= f(x_n) + d_1(y_n - x_n), \\ y_{n+1} &= f(y_n) + d_2(x_n - y_n). \end{aligned} \tag{1}$$

It is well-known that this systems can synchronize for some ranges of $d_{1,2} \in \mathscr{R}$, i.e., $|x_n - y_n| \to 0$ as $n \to \infty$ [6]. In the complete synchronized regime, the dynamics of the coupled system (1) is restricted to one-dimensional invariant subspace $x_n = y_n$, so we have the classical example of the system which is two-dimensional but the dynamics takes place on the one-dimensional attractor $A$.

The infinite number of unstable periodic orbits (UPOs) is embedded in a chaotic attractor $A$ [2]. UPOs provide the skeleton of the attractor and it allows for the characterization and the estimation in a fundamental way of many dynamical invariants. UPOs play a fundamental role in the mechanism of destabilization of the chaotic attractor localized in some symmetric invariant manifold and it is responsible for the dynamics of the phenomena such as riddling of the basin of attraction and bubbling of the chaotic attractor [5]. Recently, UPOs have also been used in the description of higher-dimensional dynamical phenomena of chaos-hyperchaos transition (i.e., transition from the attractor characterized by one positive Lyapunov exponent to the attractor characterized by at least two positive exponents) [3]. The

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simultaneous existence of UPOs with different number of unstable direction gives rise to a new kind of nonhyperbolicity known as unstable dimension variability [4] and it may give a possible dynamic mechanism for interstellar journeys.

Consider the $n$-dimensional chaotic attractor $A$ located in a $m$-dimensional phase space ($n < m$), as shown in Fig. 1. Assume that UPOs embedded in the attractor have already undergone riddling bifurcation [8] so in the attractor two types of UPOs are embedded. The first type are the orbits with stable transverse manifold (indicated in red colour in Fig. 1) and the second type these with unstable transverse manifold (indicated in green). Suppose that the phase space trajectory on the attractor $A$ is close to the point $a$ and that we have to implement the control procedure which allows us to go from the point $a$ to the point $b \in A$ (Fig. 2). The straightforward way is to restrict the path from $a$ to $b$ to the attractor $A$ (navy blue line in Fig. 2). Due to the ergodicity point $b$ will be reached in finite time but this could be too long for practical acceptance. The alternative approach is the path in the neighborhood of the attractor $A$ like the black line in Fig. 2. In this case the phase space trajectory has to leave the attractor $A$, stay in its neighborhood and return to the attractor in the appropriate point. Dynamically such a situation is possible when in the neighborhood of the point $a$ there exists UPO with unstable transverse manifold then by applying control one can move the trajectory to the neighborhood of this UPO (along the broken red line in Fig. 2) and allow it to leave the attractor (along the green line in Fig. 2). After leaving the attractor the trajectory has to come close to the stable manifold of the other UPO with stable transverse manifold (red line in Fig. 2). At this point the control moving the trajectory to the stable manifold has to be applied. Along the dashed manifold the trajectory returns to the attractor in the neighborhood of the point $b$. After another application of the control the trajectory reaches the target point $b$ (yellow line in Fig. 2).

Now recall one of the fundamental problems in the discussion of the possible interstellar journeys. Due to the huge distance in the universe and speed limitations of the space crafts, the journeys to the other stellar systems seem to be impossible (at least at the current state of science and technology) carried or even imagine. On the other hand, there are plenty of descriptions of these journeys in science-fiction literature (for example [7]). In the great number of novels such journeys are possible due to the existence of super- (or extra-) space which exists besides the universe in which human beings are living. Huge distances are covered by entering this super-space and returning to the home universe in the appropriate point. Usually there are some limitations: (i) super-space is not reachable from any point of the universe but only from the neighborhood of some special points, (ii) the energy is necessary for entering and leaving super-space, (iii) the return to the target point of the home universe is not always possible but the return in the neighborhood of the target is possible.

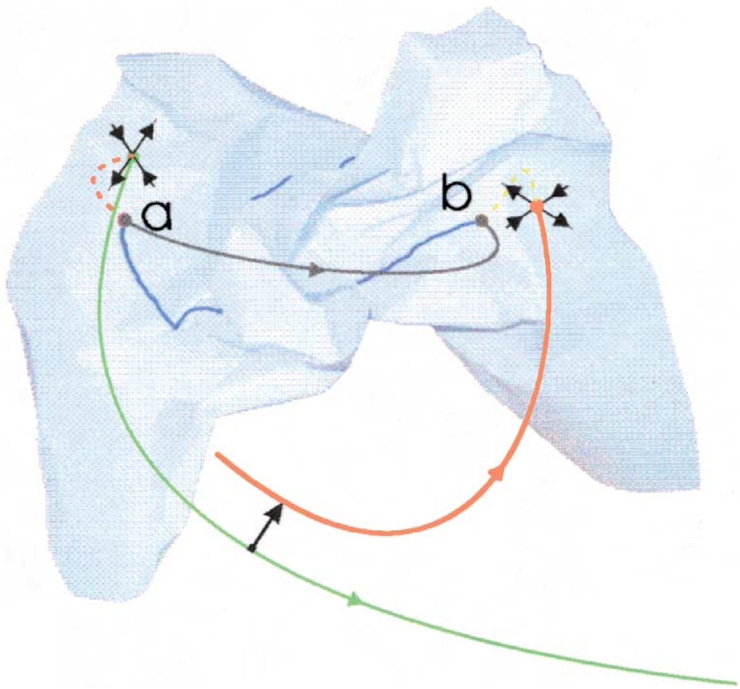


Fig. 1. Chaotic attractor $A$ with UPO with stable (red) and unstable (green) transverse manifolds.

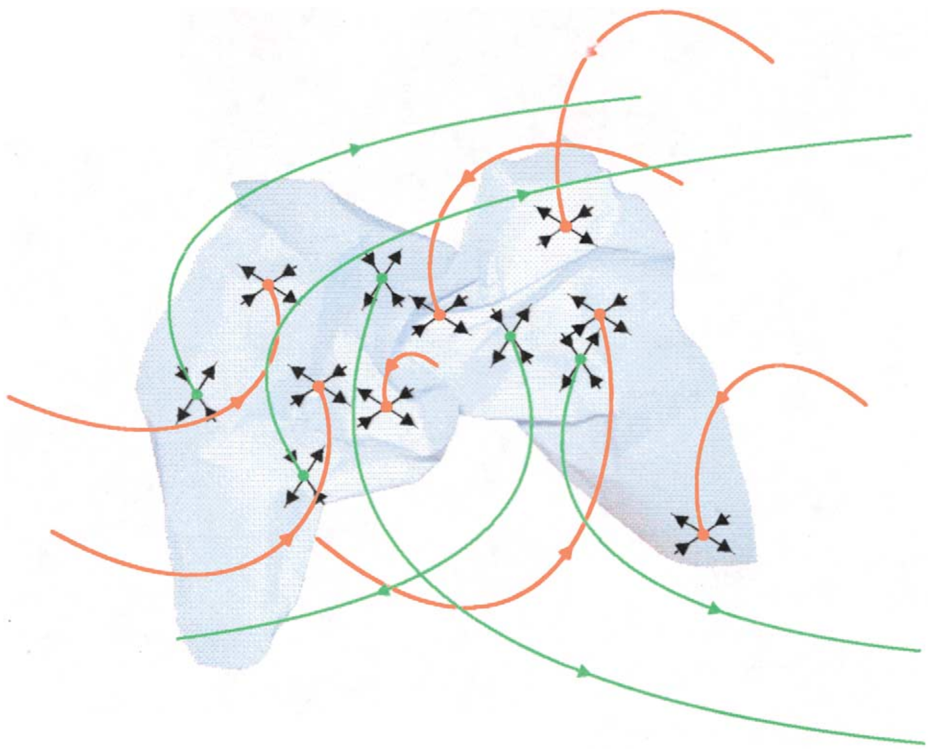

Fig. 2. Controlling procedure.

Now assume that (i) the points which allow entering super-space are UPOs with transversally unstable manifolds, (ii) the return to the universe is possible along the transversally stable manifolds of the appropriate UPO, (iii) the energy is necessary to reach the neighborhood of appropriate UPO and to change manifolds in the super-space.

At this point, one immediately finds out the analogy between the controlling procedure based on Fig. 2 and interstellar journeys (at least these described in science-fiction literature). The existence of the points in the universe which underwent riddling bifurcation (black holes,....??) is the necessary condition for interstellar journeys.

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