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Determination of geometrical conditions of assembly and impacts in classified types of mechanical systems with impacts

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Abstract

The object of investigations are systems with impacts with one and two degrees of freedom that have been classified by the authors previously. The models of the systems under consideration are rigid bodies that can perform a motion along a straight line without a possibility of rotations. The geometrical conditions of assembly and the geometrical conditions of inner and outer impacts have been determined in this study. According to the authors' viewpoint, the determination of these conditions will find an application in calculations and design of the structures described.

Keywords: Geometrical conditions of assembly; Geometrical conditions of impacts; Classification of types of impact systems; Principle of omitting the condition; Graphic method of ranges of impacts

1. Introduction

In a large number of diverse engineering fields, design or working conditions lead to collisions between the moving components of the system. This occurs when the vibration amplitudes of some components of systems (clearances or gaps) exceed critical values. The broad interest in analysing and understanding the performance of such systems is reflected by a still increasing amount of investigations devoted to this area. A few examples of such research are reported in references. Examples of this type include gears (e.g. Kaharaman and Singh, 1990; Lin and Bapat, 1993; Natsiavas, 1993), vibration isolation elements (e.g. Natsiavas, 1993), piping systems (e.g. Natsiavas, 1993), bearings (e.g. Lin and Bapat, 1993; Natsiavas, 1993), buildings during earthquakes (e.g. Nguyen et al., 1987; Nigm and Shabana, 1983), impact dampers (e.g. Bajkowski, 1996; Peterka and Blazejczyk-Okolewska, 2004), impact hammers (e.g. Fu and Paul, 1969; Tung and Show, 1988) and heat exchangers (e.g. Blazejczyk-Okolewska and Czolczynski, 1998; Goyda and The, 1980; Lin and Bapat, 1993). For a review of engineering approaches to impact systems, see also the monograph by Brogliato (1999). While analysing the studies devoted to mechanical systems with impacts, one can state that researchers have focused on systems that differ in various aspects, namely: (a) number of degrees of freedom – systems with one degree of freedom (e.g. Hinrichs et al., 1997), two degrees of freedom (e.g. Peterka and Blazejczyk-Okolewska, 2004), three degrees of freedom (e.g. Cempel, 1970), etc., (b) number of limiting stops (fenders) – with one-sided limiting stops (e.g. Hinrichs et al., 1997) or two-sided limiting stops (e.g. Peterka and Blazejczyk-Okolewska, 2004), (c) way the limiting stops displace (e.g. Peterka and Blazejczyk-Okolewska, 2004) or do not displace (e.g. Hinrichs et al., 1997) designs of the supporting structure – systems in which the supporting structures of subsystems depend on one another

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(e.g. Cempel, 1970) and systems with the subsystems that have independent supporting structures (e.g. Cempel, 1970), (e) type of forces that occur in the system – elasticity forces (e.g. Bajkowski, 1996; Bapat, 1998) and energy dissipation forces as, for instance, viscous damping forces (e.g. Peterka and Blazejczyk-Okolewska, 2004) or friction forces (e.g. Hinrichs et al., 1997), (f) number of excitations applied – to one body (e.g. Bajkowski, 1996; Hinrichs et al., 1997; Kaharaman and Singh, 1990; Lin and Bapat, 1993; Natsiavas, 1993; Nguyen et al., 1987; Nigm and Shabana, 1983; Nordmark, 1991) or to two or more bodies (e.g. Luo and Xie, 2002), (g) kind of excitation – kinematic (e.g. Lin and Bapat, 1993) or dynamic (e.g. Cempel, 1970; Peterka and Blazejczyk-Okolewska, 2004), (h) characteristics of the forces analysed in the system – elasticity forces: linear (e.g. Peterka and Blazejczyk-Okolewska, 2004) and nonlinear (e.g. Shaw and Holmes, 1983), damping forces: linear (e.g. Peterka and Blazejczyk-Okolewska, 2004) and nonlinear (e.g. Mashri, 1966), friction forces: linear (e.g. Blazejczyk-Okolewska and Kapitaniak, 1996) and nonlinear (e.g. Blazejczyk-Okolewska and Kapitaniak, 1996), (i) kind of limiting stops – rigid limiting stops (e.g. Blazejczyk-Okolewska and Kapitaniak, 1996; Nordmark, 1991) or soft limiting stops (e.g. Kaharaman and Singh, 1990; Lin and Bapat, 1993; Natsiavas, 1993; Shaw and Holmes, 1983).

In the study by Blazejczyk-Okolewska et al. (2004), a way in which subsequent types of mechanical systems with impacts arise has been presented and their classification has been shown. The study has answered the question: how many types of systems with impacts exist in general and what these types are, and has led to numerous conclusions. For instance, it has been stated that the scientific publications that include the analysis of mechanical systems with impacts and that were issued before the publication of the study by Blazejczyk-Okolewska et al. (2004) are the works that include the analysis of systems that are particular cases of the so-called basic systems (the majority of them are basic spring-impact-damping-excitation systems) proposed by the authors of the above-mentioned study.

The object of considerations in the present study are systems with one and two degrees of freedom that, according to the classification of mechanical systems with impacts (Blazejczyk-Okolewska et al., 2004), can be presented (in the case forces of elasticity and impact forces are present in the system) as basic spring-impact systems. The basic spring-impact system is a system in which each subsystem is connected to any other subsystem by means of a spring (each subsystem is also connected to a frame by means of a spring) and it impacts on any other subsystem (every subsystem impacts on the frame as well) at both possible senses of the relative velocity. Fig. 1(a) shows a basic spring-impact system for the system with one degree of freedom – it includes thus the spring k_1 that connects a body of the mass m_1 to the frame and two outer fenders z_{1g} and z_{1d} (the symbol ψ denotes that the upper fender z_{1g} occurs, whereas the symbol ζ refers to the occurrence of the lower fender z_{1d}), which make impacts of a body of the mass m_1 on the frame possible. Fig. 1(b) depicts a basic spring-impact system for the two-degree-of-freedom system – it contains thus three springs k_1, k_2, k_{12} (the spring k_1 is the spring that connects a body of the mass m_1 to the frame, the spring k_2 is the spring that connects a body of the mass m_2 to the frame, whereas the spring k_{12} is the spring that couples a body of the mass m_1 with a body of the mass m_2) and six fenders: $z_{1g}, z_{1d}, z_{2g}, z_{2d}, z_{12g}, z_{12d}$ (four outer fenders: z_{1g} and z_{1d} – that enable impacts of a body of the mass m_1 on the frame, z_{2g} and z_{2d} – that enable impacts of a body of the mass m_2 on the frame, and two inner fenders: z_{12g} and z_{12d} – that enable impacts between subsystems, that is to say, impacts of a body of the mass m_1 on a body of the mass m_2).

The discussed classification of types of mechanical systems with impacts (Blazejczyk-Okolewska et al., 2004) has a fundamental meaning in their designing process. Dimensions and the degree of complexity of some classified systems cause that the design of these new types is incurred by high risk. Even an experienced designer with extensive intuition cannot predict fully if a prototype of the system with impacts in building of which great costs and often efforts of numerous people have been involved will fulfill the foreseen assumptions, if the forecast impacts will occur in it. Maybe a fender that is the so-called passive fender

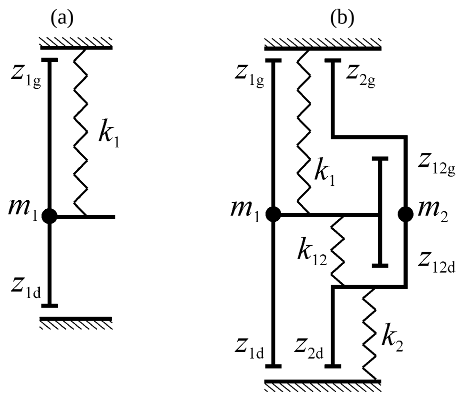

Fig. 1. Basic spring-impact systems, according to the principles of classification (Blazejczyk-Okolewska et al., 2004): (a) system with one degree of freedom, (b) system with two degrees of freedom.

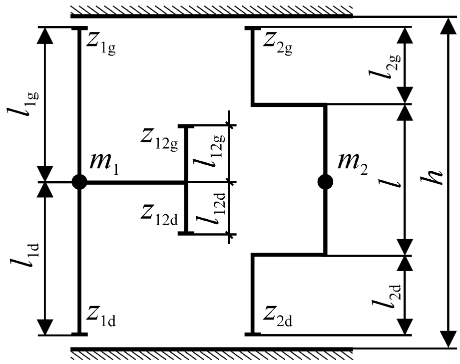

Fig. 2. Scheme of the system with corresponding notations.

which does not impact on anything (see *subphase III of Phase II* in Blazejczyk-Okolewska et al., 2004) has been assembled in the system? This aspect is especially important during calculations and design of systems with impacts.

The primary conditions that such subsystems of any system have to satisfy are the geometrical conditions that allow for the system assembly and the geometrical conditions that permit for outer and inner impacts in the system. Both the types of conditions will be determined in the study below.

2. Basic notations

Fig. 2 shows a system that consists of two subsystems, which after fulfilling certain geometrical conditions can become a system with two degrees of freedom (as in Fig. 1(b)). One subsystem is a body of the mass m_1 of the following dimensions: l_{1g} – length of the upper outer fender z_{1g}, l_{1d} – length of the lower outer fender z_{1d}, l_{12g} – length of the upper inner fender z_{12g}, l_{12d} – length of the lower inner fender z_{12d}. The second subsystem is a body of the mass m_2 of the following dimensions: l – length of the link (inner gap), in which inner impacts between masses m_1 and m_2 occur, l_{2g} – length of the upper outer fender z_{2g}, l_{2d} – length of the lower outer fender z_{2d}. The quantity h refers to the length of a gap in the frame (a distance between surfaces arbitrarily assumed as the upper and lower one in the frame). Let us notice that before assembly, the subsystem of the mass m_1 can represent a system with one degree of freedom (as in Fig. 1(a)).

3. Geometrical conditions of assembly

The geometrical conditions that allow for assembling the system are as follows:

1) gap length h in the frame longer than the sum of lengths of the fenders l_{1g} and l_{1d} (Fig. 3(a)),

$$h > l_{1g} + l_{1d}, \tag{1}$$

2) gap length h in the frame longer than the sum of lengths of the fenders l_{2g} and l_{2d} and the inner gap l (Fig. 3(b)),

$$h > l_{2g} + l_{2d} + l, \tag{2}$$

3) inner gap length l longer than the sum of lengths of the fenders l_{12g} and l_{12d} (Fig. 3(c)),

$$l > l_{12g} + l_{12d}, \tag{3}$$

4) gap length h in the frame longer than the sum of lengths of the fenders l_{1g}, l_{2d} and l_{12d} (Fig. 3(d), the first case when there is not enough room for the system after its assembly),

$$h > l_{1g} + l_{2d} + l_{12d}, \tag{4}$$

5) gap length h in the frame longer than the sum of lengths of the fenders l_{1d}, l_{2g} and l_{12g} (Fig. 3(e), the second case when there is not enough room for the system after its assembly),

$$h > l_{1d} + l_{2g} + l_{12g}. \tag{5}$$

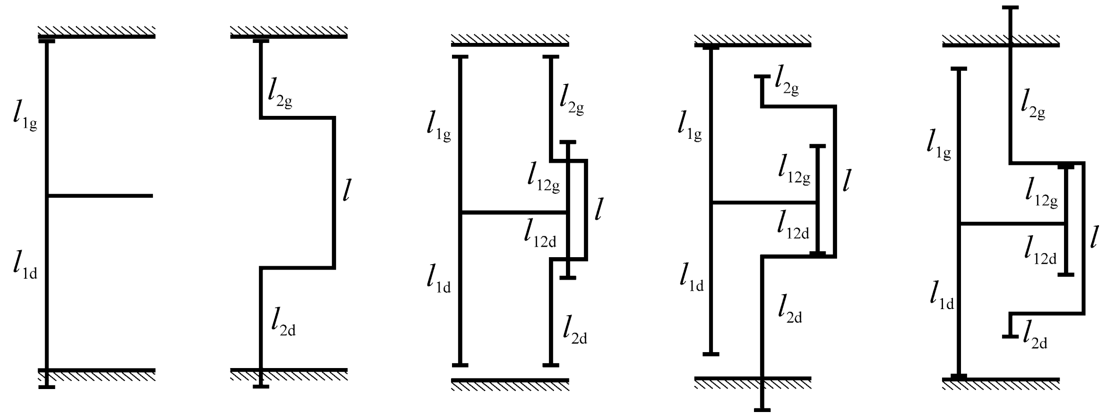

Fig. 3. Schemes showing a lack of a possibility of assembling the system – (a) inequality (1), (b) inequality (2), (c) inequality (3), (d) inequality (4), (e) inequality (5) is not satisfied, correspondingly.

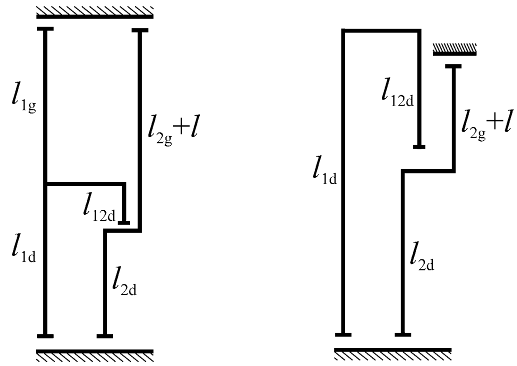

Fig. 4. Sample schemes of systems without the inner fender z_{12g}.

Each of the above-mentioned conditions has been depicted in Fig. 3. The first two conditions (Figs. 3(a) and 3(b), correspondingly) present a lack of a possibility to assemble each subsystem individually in the gap h of the frame. Thus, both the instances represent the cases that should not take place, as they do not fulfil inequalities (1) and (2). If the sum of lengths of the inner fenders l_{12g} and l_{12d} is bigger than the inner gap length l, then the system will never be assembled, either, and such a situation is depicted in Fig. 3(c). In order to connect two subsystems, inequality (3) has to be satisfied.

Let us assume that the first three conditions are met, that is to say, inequalities (1), (2) and (3) are fulfilled. However, it turns out that after assembling two subsystems in one system, it does not fit into the gap h of the frame. Such two situations where the system does not fit into the frame gap after its assembly are shown in Figs. 3(d) and 3(e). In order to avoid such a situation after assembly, the dimensions of the system have to fulfil inequalities (4) and (5).

All the above-described conditions hold in the situation when the system under consideration has all fenders, so that its dimensions can be shown in the way presented in Fig. 2. If, however, the system analysed does not have one of the fenders, then *the principle of omitting the condition* holds, according to which the condition that includes the dimension connected with this fender does not hold automatically.

This principle refers to the length of fenders only, and not to the length of gaps. For instance, let us assume that there is no inner fender z_{12g} in the system, and thus there is no dimension l_{12g}, then the geometrical conditions of assembly reduce to the three conditions that can be described by inequalities (1), (2) and (4). An inquiring reader may ask a question: So what that there is no fender z_{12g} (dimension l_{12g}), if – with a relative motion of two subsystems – an impact in the position of the lacking fender occurs, then what are these conditions for? The answer is as follows: the designer should design this connection in such a way that an impact will not occur in this position. For example, in the position of the lacking fender, two subsystems could omit simply each other and never impact on each other at all. An equally good answer follows from Fig. 4(a), where the dimension l_{2g} of the upper fender is equal to the sum $l_{2g} + l$, and there is no gap in the system. Then, we have the following dimensions: for a body of the mass $m_1 - l_{1g}, l_{1d}, l_{12d}$, for a body of the mass $m_2 - l_{2d}, l_{2g} + l$. On the other hand, another reader can say: OK, but on the assumption that there is no, for instance, fender z_{1g} (dimension l_{1g}) in the system, conditions (1) and (4) are omitted. However, should not condition (1) be satisfied, if we assume that $l_{1g} = 0$? The answer is as follows: No, it is unnecessary, because in this particular case, we have still condition (5), which holds for $h > l_{1d}$. Moreover, an example of the system shown in Fig. 4(b) proves that the dimension l_{1d} does not have to be included in h.

The geometrical conditions of assembly have been presented in such a way that they hold for all, previously classified systems with one and two degrees of freedom. In the case of a system with one degree of freedom (there is no subsystem of the mass m_2), according to the principle of omitting the condition, condition (1) holds only.

4. Geometrical conditions of impacts

In the systems under consideration (Fig. 1), impacts can occur when the defined conditions are met. Impacts have been divided into two groups: outer impacts and the inner ones.

4.1. Geometrical conditions of outer impacts

By outer impacts are meant impacts on the frame, in which the fenders z_{1g}, z_{2g}, z_{1d} and z_{2d} (Fig. 1 or Fig. 2) take part.

The geometrical conditions that have to be satisfied by the system dimensions and that enable outer impacts are following:

1) in order for the fender z_{1g} to impact on the frame, the fender length l_{1g} has to be longer than the sum of the lengths $l_{2g} + l_{12g}$ of the fenders z_{2g} and z_{12g} (Fig. 5(a)), respectively,

$$l_{1g} > l_{2g} + l_{12g}, \tag{6}$$

2) in order for the fender z_{2g} to impact on the frame, the sum of the fender length l_{2g} and the inner gap l has to be bigger than the sum of the lengths l_{1g} and l_{12d} of the fenders z_{1g} and z_{12d} (Fig. 5(b)), respectively,

$$l_{2g} + l > l_{1g} + l_{12d}, \tag{7}$$

3) in order for the fender z_{1d} to impact on the frame, the fender length l_{1d} has to be longer than the sum of the lengths l_{2d} and l_{12d} of the fenders z_{2d} and z_{12d} (Fig. 5(c)), respectively,

$$l_{1d} > l_{2d} + l_{12d}, \tag{8}$$

4) in order for the fender z_{2d} to impact on the frame, the sum of the fender length l_{2d} and the inner gap length l has to be bigger than the sum of the lengths l_{1d} and l_{12g} of the fenders z_{1d} and z_{12g} (Fig. 5(d)), respectively,

$$l_{2d} + l > l_{1d} + l_{12g}. \tag{9}$$

Figs. 5(a)–(c) and 5(d) show situations when outer impacts of the fenders z_{1g}, z_{2g}, z_{1d} and z_{2d}, respectively, will not take place in the system. For example, Fig. 5(a) depicts a case when the fender z_{1g} will never be able to impact on the frame. The obstacle lies in too high a value of the sum of lengths of the fenders z_{2g} and z_{12g}, and, strictly speaking, the blocking of the

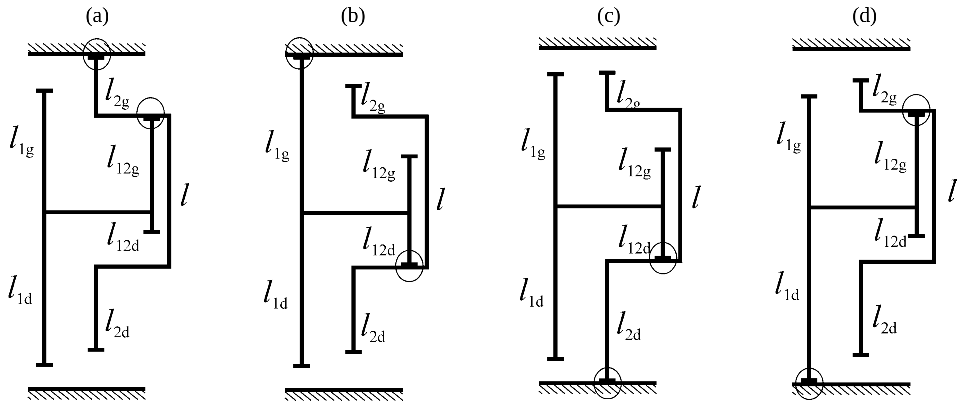

Fig. 5. Schemes of the systems in which outer impacts will never occur.

upward motion of the subsystem of the mass m_1 by the inner fender z_{12g}. In order to prevent such a situation, condition of inequality (6) has to be met.

In the case of the geometrical conditions of outer impacts, *the principle of omitting the condition* holds, like for the assembly conditions. If the system under consideration does not have one of outer fenders, then the condition that contains the dimension connected with this fender is omitted automatically. For instance, let us assume that the system does not have the outer fender z_{1g}, and thus it does not have the dimension l_{1g}, then the geometrical conditions of outer impacts of such a system assume the form of two inequalities only, namely inequality (8) and (9). An inquiring reader will say instantly: How is it? There is no fender z_{1g}, and the condition for outer impacts with the fender z_{2g} (that is to say, inequality (7)) is omitted? Yes, it is so because inequality (7) is then satisfied automatically. When there is no fender z_{1g}, then an outer impact with the fender z_{2g} will always occur, if such a fender exists of course.

The geometrical conditions of outer impacts have been presented in such a way that they hold for all, previously classified types of systems with one and two degrees of freedom. In the case of a one-degree-of-freedom system (no subsystem of the mass m_2), outer impacts always occur (as long as the fenders z_{1g} and z_{1d} exist), despite the fact that, according to *the principle of omitting the condition*, conditions (6)–(8) and (9) are omitted.

After the analysis conducted in such a way, a question arises if it is possible to determine such conditions that define explicitly if outer impacts occur in a given system and what impacts take place. Will – in a real, assembled system – all outer impacts occur or only three or two selected ones or maybe just one impact? This question can be answered comprehensively thanks to the *graphic method of ranges of outer impacts*, which has been developed by the authors and which consists in plotting a diagram with ranges of outer impacts in the system.

4.2. Graphic method of ranges of outer impacts

Fig. 6 shows a diagram of ranges of outer impacts. This is a diagram that has been referred only to two dimensions of the subsystem of the mass m_1, namely the dimension l_{1g} and the dimension l_{1d}, i.e., the dimensions that are connected with outer fenders (z_{1g} and z_{1d}) of this subsystem.

Let us notice that both the dimensions, i.e. l_{1g} and l_{1d}, on the diagram are directly connected with the first geometrical condition of assembly, which informs us about the fact that the sum of the link lengths $l_{1g} + l_{1d}$ has to be smaller than the length of the gap h of the frame. Owing to this fact, *the basic straight line* (thickened line), described by the equation $l_{1d} = h_1 - l_{1g}$ ($h_1 = l_{1g} + l_{1d}$), has been drawn on the diagram, and any point taken from the region confined by it or belonging to it always fulfils the inequality $l_{1g} + l_{1d} \leqslant h_1$.

The points of the basic straight line $l_{1d} = h_1 - l_{1g}$ that intersect the axes l_{1g} and l_{1d} are the points: on the axis l_{1g} – value $l_{1g} = h_1$ ($l_{1d} = 0$), on the axis l_{1d} – value $l_{1d} = h_1$ (for $l_{1g} = 0$). Both the values (the ultimate ones) are equal, which makes this straight line directed at the angle of 45° with respect to both the axes of the diagram.

Apart from this, the diagram presents all conditions for outer impacts. The first two of them (6) and (7) are located on the horizontal axis l_{1g}. As the value $l_{2g} + l_{12g}$ from condition (6) is lower than the value $l_{2g} + l - l_{12d}$ from condition (7) (because $l - l_{12d} > l_{12g} \Rightarrow l > l_{12g} + l_{12d}$, the third condition of assembly (3)), we place it as the closer one to "0" on the axis l_{1g}, and next, the second one of them. Thus, the range $0 < l_{1g} < h_1 - l_{1d}$ of the diagram we are interested in has been divided into three subregions, namely: $0 < l_{1g} < l_{2g} + l_{12g}, l_{2g} + l_{12g} < l_{1g} < l_{2g} + l - l_{12d}, l_{2g} + l - l_{12d} < l_{1g} < h_1 - l_{1d}$. We act analogously with the two remaining conditions of outer impacts (8) and (9). The first value of them, $l_{2d} + l_{12d}$ is lower than the second value $l_{2d} + l - l_{12g}$ (because $l_{12d} < l - l_{12g} \Rightarrow l > l_{12d} + l_{12g}$, the third condition of assembly (3)), we place it as the closer one to "0" on the axis l_{1d}, and next, the second one of them. Thus, the range $0 < l_{1d} < h_1 - l_{1g}$ we are interested in on the plot has been divided into three subregions, namely: $0 < l_{1d} < l_{2d} + l_{12d}, l_{2g} + l_{12d} < l_{1d} < l_{2d} + l - l_{12g}, l_{2d} + l - l_{12g} < l_{1d} < h_1 - l_{1d}$.

The diagram prepared in this way is composed of nine rectangular regions (region A: 1-2-7-8, region B: 2-3-6-7, region C: 3-4-5-6, region D: 8-7-10-9, region E: 7-6-11-10, region F: 6-5-12-11, region G: 9-10-15-16, region H: 10-11-14-15, region I: 11-12-13-14) that describe various possibilities of outer impacts. Each upper left corner of the region, filled with the notation z_{1g} (regions: B, C, E, F, H, I), informs us about a possibility of occurrence of an outer impact of the upper subsystem of the mass m_1 on the frame. Each lower left corner of the region, filled with the notation z_{1d} (A, B, C, D, E, F), informs us about a possibility of occurrence of an outer impact of the lower subsystem of the mass m_1 on the frame. Each upper right corner of the region, filled with the notation z_{2g} (regions: A, B, D, E, G, H), informs us about a possibility of occurrence of an outer impact of the upper subsystem of the mass m_2 on the frame. Each lower right corner of the region, filled with the notation z_{2d} (regions: D, E, F, G, H, I), informs us about a possibility of occurrence of an outer impact of the lower subsystem of the mass m_2 on the frame.

It follows from this diagram that in order to, for instance, have all cases of outer impacts ($z_{1g}, z_{2g}, z_{1d}, z_{2d}$), we have to be in the region E, where the following conditions on the lengths of the fenders l_{1g} and l_{1d}: $l_{2g} + l_{12g} < l_{1g} < l_{2g} + l - l_{12g}$ and $l_{2d} + l_{12d} < l_{1d} < l_{2d} + l - l_{12g}$ are fulfilled. Having established their values, we have to remember that the parameter h has to be higher than $h_1 = l_{1g} + l_{1d}$, according to the previous assumptions. It can be the value, e.g., $h = h_E$ (then the point a_E of the

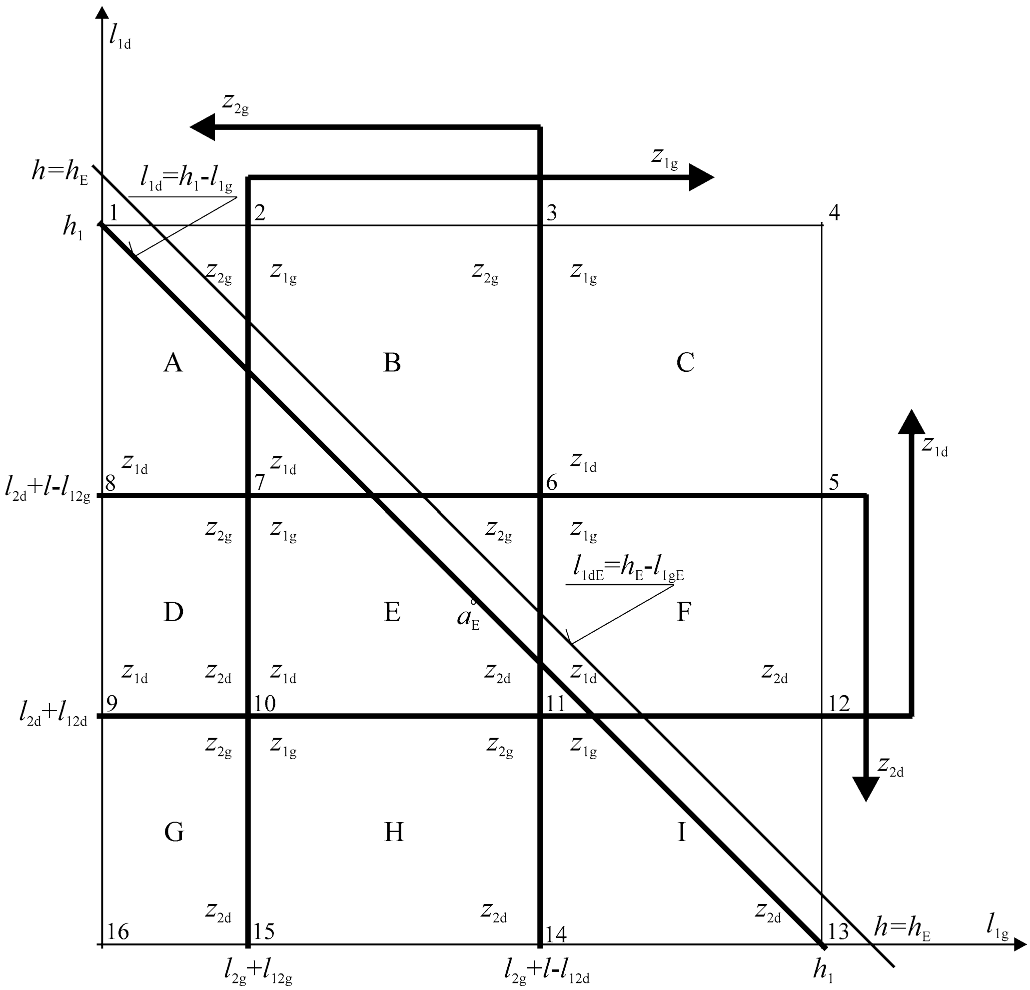

Fig. 6. Diagram of the ranges of outer impacts, applied in the graphic method.

region E lies below the straight line described by the equation $l_{1dE} = h_E - l_{1gE}$). It is also possible to have all outer impacts, i.e., to be in the region E without increasing the parameter h. We assume then that $h = h_1 = \text{const}$ and, for instance, $l_{1d} = \text{const}$, and the value of the parameter l_{1g} has to satisfy the inequality $l_{1g} < h_1 - l_{1d}$ (l_{1g} and l_{1d} must be values from the region E).

The structure of the diagram becomes simple when one of the fenders is lacking. For instance, when there is no fender z_{1g}, that is to say, the dimension l_{1g}, only the axis l_{1d} of the diagram holds, with the ranges of conditions for outer impacts that hold on it (see Example 2 in Section 5, where examples of application of the diagram of ranges of outer impacts are given and their advantages are described).

4.3. Geometrical conditions of inner impacts

By inner impacts we mean impacts between the subsystem of the mass m_1 and the subsystem of the mass m_2, that is say, such impacts in which the fenders z_{12g} and z_{12d} (Figs. 1 or 2) take part.

The geometrical conditions that the system dimensions have to fulfil and that enable inner impacts are as follows:

1) in order for the subsystem of the mass m_1 to impact on the subsystem of the mass m_2 with the fender z_{12g}, the following inequality has to be satisfied (Fig. 7(a)),

$$(l_{1g} - l_{12g}) + (l + l_{2d}) < h, \tag{10}$$

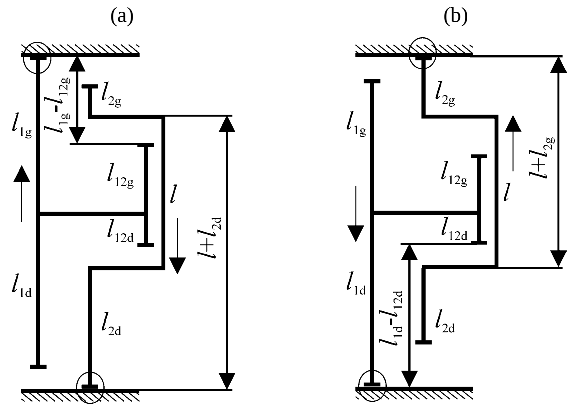

Fig. 7. Schemes of the systems in which inner impacts will never occur.

2) in order for the subsystem of the mass m_1 to impact on the subsystem of the mass m_2 with the fender z_{12d}, the following inequality has to be satisfied (Fig. 7(b)),

$$(l_{1d} - l_{12d}) + (l + l_{2g}) < h. \tag{11}$$

Fig. 7 shows a situation when inner impacts performed by the fenders z_{12g} and z_{12g} will never occur in the system.

Fig. 7(a) presents a case when the fender z_{12g} will never be active during the relative motion of the subsystem of the mass m_1 upwards and the subsystem of the mass m_2 downwards. An obstacle lies in too high a value of the sum that consists of the addends denoted in the figure by $l_{1g} - l_{12g}$ and $l + l_{1d}$. This sum is bigger than the length of the gap h of the frame. The activity of this fender is thus limited due to a lack of a possibility of further displacement of the subsystems with respect to each other. Fig. 7(b) is a case when the fender z_{12d} will never be active during the relative motion of the subsystem of the mass m_1 downwards and the subsystem of the mass m_2 upwards. An obstacle lies in too high a value of the sum that consists of the addends denoted in the figure by $l_{1d} - l_{12d}$ and $l + l_{2g}$. This sum is bigger than the length of the gap h of the frame. The activity of this fender is thus also limited due to a lack of a possibility of further displacement of the subsystems with respect to each other.

If the system under consideration does not have one inner fender, then *the principle of omitting the condition*, according to which the condition, which includes the dimension related to this fender does not hold automatically, acts as well.

The geometrical conditions of inner impacts have been presented in such a way that they hold for all, previously classified systems with two degrees of freedom. In the case of a system with one degree of freedom (there is no subsystem of the mass m_2), we do not consider inner impacts.

In the case of outer impacts, it was possible to prepare a general diagram of ranges of impacts. However, in the case of inner impacts, the situation becomes slightly more complex. It is difficult to place the values of conditions for inner impacts, that is to say: $l_{1g} < h - (l + l_{2d} - l_{12g})$ and $l_{1d} < h - (l + l_{2g} - l_{12d})$, on the diagram from Fig. 6. For a particular case, as in Examples 1 and 2 of Subsections 5.1 and 5.2, respectively, the situation becomes much simpler. There, the examples of systems with the diagrams of ranges of outer and inner impacts have been given. It can be thus concluded that the diagram of impacts from Subsection 4.2 plays a very useful role. Having a particular system with given dimensions, we can define what impacts will occur in the system and which fenders should have their lengths changed to have another set of outer and inner impacts.

In all conditions for outer and inner impacts, sharp inequalities occur. In the case we write these conditions in the form of non-sharp inequalities, a situation arises in which outer impacts can occur simultaneously or outer and inner impacts can occur simultaneously.

5. Examples of applications of the diagram with ranges of impacts – discussion of the results

The examples presented in this section are aimed not only to examine the introduced geometrical conditions of assembly and impacts, but also to indicate a possibility of employment of the above-discussed method, that is to say, the graphic method of ranges of impacts, in practical applications.

5.1. Example 1

For the system presented in Fig. 2, check a possibility of assembly and occurrence of outer and inner impacts, on the assumption of the following dimensionless lengths: $l_{1g} = 10, l_{1d} = 3, l_{12g} = 3, l_{12d} = 1, l_{2g} = 4, l = 5, l_{2d} = 4, h = 17$.

Solution:

I. Checking assembly conditions (1)–(5):

(1) $13 = l_{1g} + l_{1d} < h = 17$,

(2) $13 = l_{2g} + l + l_{2d} < h = 17$,

(3) $4 = l_{12g} + l_{12d} < l = 5$,

(4) $15 = l_{1g} + l_{2d} + l_{12d} < h = 17$,

(5) $10 = l_{1d} + l_{2g} + l_{12g} < h = 17$.

All the assembly conditions are met. Otherwise, the designer would have to alter the lengths of the fenders or the lengths of the gaps.

II. Conditions for outer impacts (6)–(8):

(6) $7 = l_{2g} + l_{12g} < l_{1g} = 10$,

(7) $8 = l_{2g} + l - l_{12d} \ngtr l_{1g} = 10$,

(8) $5 = l_{2d} + l_{12d} \nless l_{1d} = 3$,

(9) $6 = l_{2d} + l - l_{12g} > l_{1d} = 3$.

In the case of outer impacts, not all conditions are satisfied. In the system where the dimensions are assigned in this way, never all outer fenders will be active. Despite the fact that the designer assembled all outer fenders, two of them, namely: z_{2g} and z_{1d}, would always be passive (conditions (7) and (8) are not fulfilled).

III. Conditions for inner impacts (10) and (11):

(10) $16 = (l_{1g} - l_{12g}) + (l + l_{2d}) < h = 17$,

(11) $11 = (l_{1d} - l_{12d}) + (l + l_{2g}) < h = 17$.

As regards inner impacts, all conditions are met. In the system with the dimensions assigned in this way, both the fenders z_{12g} and z_{12d} will be active.

In order to place the geometrical conditions for inner impacts on the diagram of ranges of impacts, they have been written in the way that determines the range of variables on the axes l_{1g} and l_{1d}, i.e.: $l_{1g} < h - (l + l_{2d} - l_{12g}) = 11$ and $l_{1d} < h - (l + l_{2g} - l_{12d}) = 9$.

Fig. 8 shows a diagram with ranges of outer and inner impacts. The assigned data cause, as has already been mentioned, that the system after assembly can perform only two outer impacts (due to $l_{1g} = 10$ and $l_{1d} = 3$, we are in region I). They are impacts of the subsystem of the mass m_1 with the upper fender z_{1g} and impacts of the subsystem of the mass m_2, but with the lower fender z_{2d}. It also results from the diagram that the system can perform both inner impacts (the ranges of inner impacts are marked with a dashed line on the diagram).

On the diagram, the straight line that fulfils the equation $l_{1d} = h_1 - l_{1g}$, where $h_1 = l_{1g} + l_{1d} = 13$, is drawn. Let us notice that the value of the parameter h_1 plays the role of the boundary parameter if we want to enforce other sets of outer impacts in the system.

Changing the lengths of the fenders l_{1g} and l_{1d} in such a way that the chosen values (l_{1gn} and l_{1dn}) belong to the region below the straight line $l_{1d} = h_1 - l_{1g}$, and not altering the remaining assigned values $h, l_{12g}, l_{12d}, l_{2g}, l$ and l_{2d} at all, we can have various sets of outer and inner impacts through a selection of the region A, D, G, E, H or I. It is possible owing to the fact that the sum of the lengths $l_{1gn} + l_{1dn}$ will never exceed h_1 and always $h_1 < h$. The additional information is the fact that while changing the values l_{1g} and l_{1d}, the ranges of inner impacts are displaced. It follows from this that in the regions D, G, E and H, inner impacts will always take place and outer impacts defined by the conditions will occur as well (for the region D – with the fenders z_{2d}, z_{1d} and z_{2g}, for the region G – with the fenders z_{2d} and z_{2g}, for the region E – with the fenders z_{1g}, z_{2d}, z_{1d} and z_{2g}, for the region H – with the fenders z_{1g}, z_{2d} and z_{2g}). In the case of the regions A and I, while changing the values of the lengths of the fenders l_{1g} and l_{1d} into (now we consider consequently the values from the region below the straight line $l_{1d} = h_1 - l_{1g}$) the values l_{1gn} and l_{1dn}, a situation becomes slightly more complex. In the region A, outer impacts with the fenders z_{1d} and z_{2g} can occur and both inner impacts can take place when $l_{2d} + l - l_{12g} < l_{1dn} < h - (l + l_{2g} - l_{12d})$ and $0 < l_{1gn} < l_{2g} + l_{12g}$

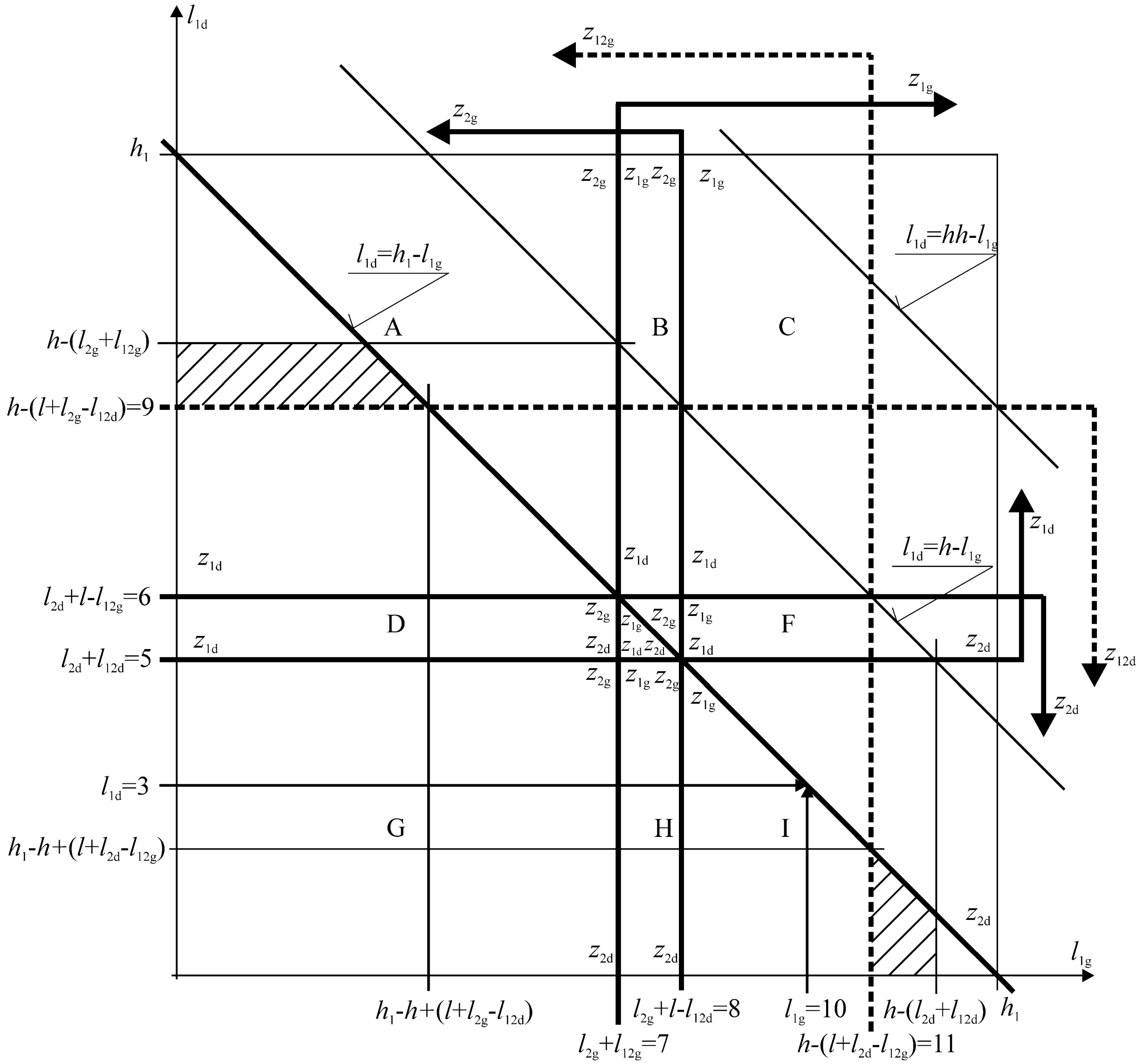

Fig. 8. Diagram of the ranges of inner impacts for Example 1 with the ranges of inner impacts.

(the area below the straight line $l_{1\mathrm{d}} = h_1 - l_{1\mathrm{g}}$ of the region A under the dashed region), or only inner impacts with the upper fender $z_{12\mathrm{g}}$ can occur when $h - (l_{2\mathrm{g}} + l_{12\mathrm{g}}) > l_{1\mathrm{dn}} > h - (l + l_{2\mathrm{g}} - l_{12\mathrm{d}})$ and $0 < l_{1\mathrm{gn}} < h_1 - h + (l + l_{2\mathrm{g}} - l_{12\mathrm{d}})$ (dashed area of the region A, in which the lower fender $z_{12\mathrm{d}}$ is passive). In the region I, outer impacts with the fenders $z_{1\mathrm{g}}$ and $z_{2\mathrm{d}}$ can occur, or both inner impacts can take place when $l_{2\mathrm{g}} + l - l_{12\mathrm{d}} < l_{1\mathrm{gn}} < h - (l + l_{2\mathrm{d}} - l_{12\mathrm{g}})$ and $0 < l_{1\mathrm{dn}} < l_{2\mathrm{d}} + l_{12\mathrm{d}}$ (the area below the straight line $l_{1\mathrm{d}} = h_1 - l_{1\mathrm{g}}$ of the region I on the left-hand side of the dashed region), or only inner impacts with the lower fender $z_{12\mathrm{d}}$ can occur if $h - (l + l_{2\mathrm{d}} - l_{12\mathrm{g}} < l_{1\mathrm{gn}} < h - (l_{2\mathrm{d}} + l_{12\mathrm{d}})$ and $0 < l_{1\mathrm{dn}} < h_1 - h + (l + l_{2\mathrm{d}} - l_{12\mathrm{g}})$ (dashed area of the region I, in which the upper fender $z_{12\mathrm{g}}$ is passive).

For $l_{1\mathrm{dn}} > h - (l_{2\mathrm{g}} + l_{12\mathrm{g}})$ of the region A lying below the straight line $l_{1\mathrm{d}} = h_1 - l_{1\mathrm{g}}$ (the boundary value of one of the conditions), assembly condition (5) is not met. For $l_{1\mathrm{gn}} > h - (l_{2\mathrm{d}} + l_{12\mathrm{d}})$ of the region I lying below the straight line $l_{1\mathrm{d}} = h_1 - l_{1\mathrm{g}}$ (the boundary value of one of the conditions), assembly condition (4) is not fulfilled. There is, however, a possibility of exceeding these values, for instance, through an increase in the value of the parameter h (or alternations of another parameter). While introducing changes in the values of the system parameters, we should however remember that the assembly conditions should always be satisfied and that if we change any values, then the ranges of conditions of inner impacts change as well.

Some very significant conclusions follow from the analysis conducted. First of all, it is possible to design a system in such a way that both inner fenders (z_{12g} and z_{12d}) will occur in it but only one will be active. Then a question arises: What is a passive fender assembled for? Of course, it is unnecessary. The designer should not assemble passive fenders and it should be remembered that if the system under consideration does not have one of the inner fenders, then the condition that includes the dimension related to this fender does not hold automatically, that is to say, *the principle of omitting the condition* holds. Secondly, it is impossible to design a system in such a way that both inner fenders z_{12g} and z_{12d} will occur in it and both of them will be passive (the intersection point of the dashed lines that describe the ranges of inner impacts always lies above the straight line $l_{1d} = h - l_{1g}$). In the case of undesirable inner impacts, the system should be designed so that it does not have inner fenders. Then, in the system under consideration, the conditions that include dimensions related to the fenders z_{12g} and z_{12d} do not hold automatically.

Also, let us notice (and this is an important advantage) that in all the described regions below the straight line $l_{1d} = h_1 - l_{1g}$ (D, G, E, H, A only for $l_{1d} < h - (l_{2g} + l_{12g})$ or I only for $l_{1g} < h - (l_{2d} + l_{12d})$, it is possible to minimise the value of h. However, it should be remembered that $h > h_1$ and that all the conditions of assembly should be met (in this case, for instance, $h = 15.5$) and that if this value is changed, then the ranges of conditions of inner impacts change as well.

The diagram of ranges of impacts has also such an advantage that it can be applied knowingly when it is necessary to use the values l_{1gn} and l_{1dn} from the region between the straight lines $l_{1d} = h_1 - l_{1g}, l_{1d} = h - l_{1g}$ or from the regions (even) above the straight line $l_{1d} = h - l_{1g}$. In the first case, if we select the values of l_{1gn} and l_{1dn} from the regions A, B, C, E, F and I, and remain absolutely the value $h = 17$ constant, we should remember that the assembly conditions have to be met. In the second case (regions A, B, C, F and I), the inequality $l_{1gn} + l_{1dn} > h$ holds, and thus the value h should be changed into another value, e.g. hh, such that $hh > h$ and that the assembly conditions should always be satisfied. We should always remember that if we alter the value h, the ranges of the conditions of inner impacts change and the intersection point of the dashed lines (ranges of inner impacts) will be above the straight line $l_{1d} = hh - l_{1g}$.

Let us assume that in the above-considered system, the outer impacts z_{2g} are additionally desirable, so we want to bring the system to the situation from the region H, i.e., to impacts with the fenders z_{2g} and z_{2d}. To achieve this, we change the fender length l_{1g} into a value from this region. Let it be, for instance, $l_{1gH} = 7.5$. The remaining parameters are left unchanged. Let us notice that we do not have to change the length of the fender z_{1d} ($z_{1dH} = z_{1d} = 3$) either, as this value is included in this region. In this case, we do not have to check the assembly conditions as the whole region H is situated below the straight line $l_{1d} = h_1 - l_{1g}$, and $h_1 < h$ (here h can be decreased to h_1, if necessary). To make sure, we can check the correctness of the assumptions concerning an occurrence of three outer impacts from this region:

(6) $7 = l_{2g} + l_{12g} < l_{1gH} = 7.5,$

(7) $8 = l_{2g} + l - l_{12d} > l_{1gH} = 7.5,$

(8) $5 = l_{2d} + l_{12d} \nless l_{1dH} = 3,$

(9) $6 = l_{2d} + l - l_{12g} > l_{1dH} = 3.$

The calculated conditions point out to the correctness of the assumptions concerning an occurrence of three outer impacts. The following outer fenders: z_{1g}, z_{2g} and z_{2d} will be active in the system and both inner impacts will occur (we do not need to check the conditions for inner impacts either, as $l_{1gH} < l_{1g}$).

Next, let us assume that in the system under consideration all outer impacts are desirable, and thus we want to bring the system to the situation from the region E, i.e., z_{1g}, $z_{2g}z_{1d}$ and z_{2d}. In this case, we change the fender lengths l_{1g} and l_{1d} into the values, e.g. $l_{1gE} = 7.5$ and $l_{1dE} = 5.5$. Let us notice that in this case, we do not have to check the conditions of assembly. Although $l_{1gE} + l_{1dE} = h_1$, but $h_1 < h$ (here h cannot be decreased to h_1) and the whole region E is situated below the straight line $l_{1d} = h - l_{1g}$. To make sure, we can check the correctness of the assumptions concerning an occurrence of all three outer impacts from this region:

(6) $7 = l_{2g} + l_{12g} < l_{1gE} = 7.5,$

(7) $8 = l_{2g} + l - l_{12d} > l_{1gE} = 7.5,$

(8) $5 = l_{2d} + l_{12d} < l_{1dE} = 5.5,$

(9) $6 = l_{2d} + l - l_{12g} > l_{1dE} = 5.5.$

The fulfilment of the above-mentioned conditions points out to the correctness of the assumptions made. In the system, all outer fenders will be active and both inner impacts will take place.

5.2. Example 2

For the system shown in Fig. 4(b), check a possibility of assembly and occurrence of outer and inner impacts, on the assumption of the following dimensionless lengths: $l_{1\mathrm{d}} = 10, l_{12\mathrm{d}} = 3, l_{2\mathrm{g}} + l = 2, l_{2\mathrm{d}} = 4, h = 8$.

Solution:

I. Checking the conditions of assembly:

(2) $\quad 6 = l_{2\mathrm{g}} + l + l_{2\mathrm{d}} < h = 8.$

Assembly conditions (1), (3), (4) and (5) do not hold owing to a special structure of the system, whereas condition (2) is satisfied.

II. Conditions for outer impacts:

(8) $\quad 7 = l_{2\mathrm{d}} + l_{12\mathrm{d}} < l_{1\mathrm{d}} = 10.$

In the case of outer impacts, conditions (6), (7) and (9) are omitted. Condition (8) is fulfilled. In the system with the lengths of the fenders assigned in this way, outer impacts with the lower fender $z_{1\mathrm{d}}$ will take place. Despite the fact that conditions (7) and (9) for outer impacts $z_{2\mathrm{g}}$ and $z_{2\mathrm{d}}$ are omitted, these impacts will always occur in it, owing to a special structure of the system (a lack of the fenders that could prevent it, see Figs. 5(b) and 5(d)).

III. Conditions for inner impacts:

(11) $\quad 9 = (l_{1\mathrm{d}} - l_{12\mathrm{d}}) + (l + l_{2\mathrm{g}}) \nless h = 8.$

In the case of inner impacts, condition (10) is omitted, whereas condition (11) is not met. In the system with the lengths of the links assigned in this way, inner impacts with the fender $z_{12\mathrm{d}}$ will not occur. Although there is no parameter $l_{1\mathrm{g}}$ in the system, but in Fig. 9 it has been treated as the dimension equal to zero and the straight line $l_{1\mathrm{d}} = h_1 - l_{1\mathrm{g}}$ has been drawn. We should remember, however, that in this case only the vertical axis holds (for the example given, the range D of this axis).

Let us notice that here the value of the parameter h_1 does not play the part of the boundary parameter if we want to enforce other sets of impacts in the system. Owing to a special structure of the system, h_1 does not have to be smaller than h, as the

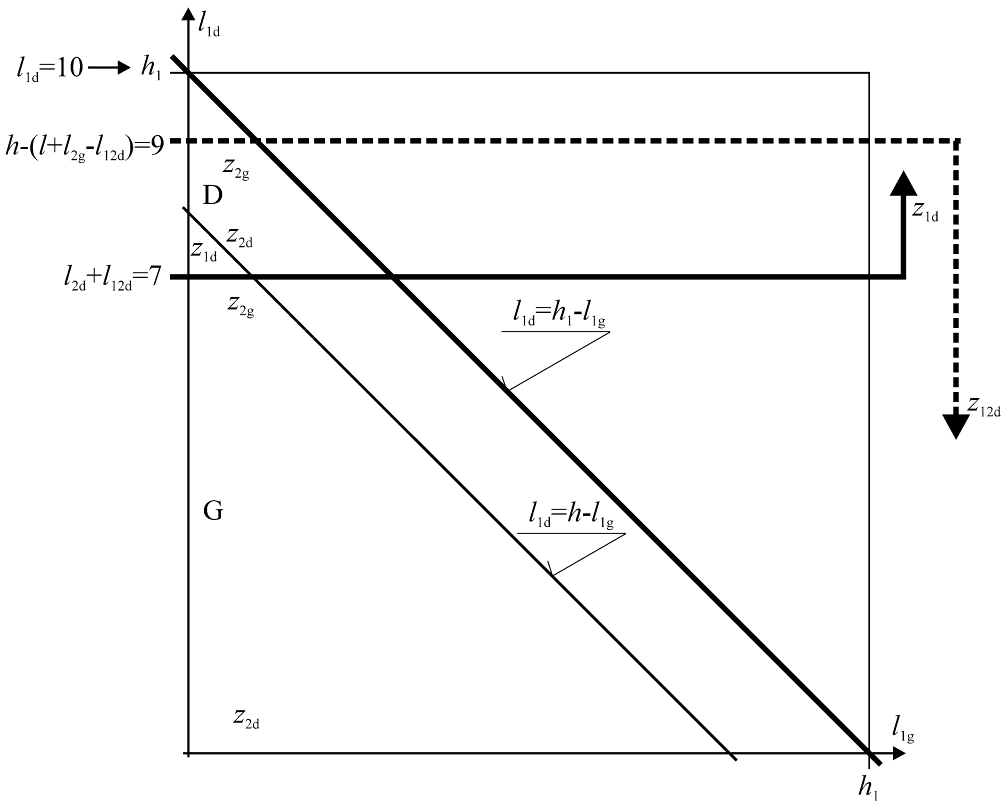

Fig. 9. Diagram of the ranges of outer impacts for Example 2.

fender length l_{1d} is not included in the gap h. If we change the fender length l_{1d} into the length l_{1dn} such that $l_{1dn} > l_{1d}$, we do not cause any changes in the behaviour of the system. We observe the same set of impacts, that is to say, outer impacts with the lower fender z_{1d} and outer impacts of the second subsystem with the fenders z_{2g} and z_{2d}. We observe the same situation if we change the length l_{1d} into the length l_{1dn} from the range $l_{1d} > l_{1dn} > h - (l + l_{2g} - l_{12d})$. However, if we decrease the length l_{1d} to the length l_{1dn} from the range $l_{2d} + l_{12d} < l_{1dn} < h - (l + l_{2g} - l_{12d})$, then apart from outer impacts with the fenders z_{1d}, z_{2d} and z_{2g}, also inner impacts can be observed with the fender z_{12d}, which stops being passive (the system designed by the designer is fully used now – all fenders are active; we are still in the range D). Let us notice also that when we decrease l_{1d} to the length l_{1dn} from the range $0 < l_{1dn} < l_{2d} + l_{12d}$, then the fender z_{1d} stops being active (range G). In the system, both outer impacts take place with the second subsystem and inner impacts occur with the fender z_{12d}.

5.3. Discussion of the examples

Both in Example 1 and 2 (Subsections 5.1 and 5.2, respectively), the possibilities of introducing changes in sets of impacts due to changes in the link lengths l_{1d} and l_{1g} have been presented.

Now, let us notice that in Example 2 (Subsection 5.2), if we want to make the inner fender active, it might be easier to change its length. It is enough to increase its length l_{12d} to the value $l_{12dn} = 5$. After this change, condition for an outer impact (8) is still satisfied:

(8) $9 = l_{2d} + l_{12dn} < l_{1d} = 10.$

If the condition for an inner impact:

(11) $7 = (l_{1d} - l_{12dn}) + (l + l_{2g}) < h = 8$

holds, then the fender z_{12dn} becomes active. At such a slight correction of the length of one of the links in the system, the outer impacts z_{1d}, z_{2g}, z_{2d} still occur and the inner impact z_{12dn} takes place as well (Fig. 10).

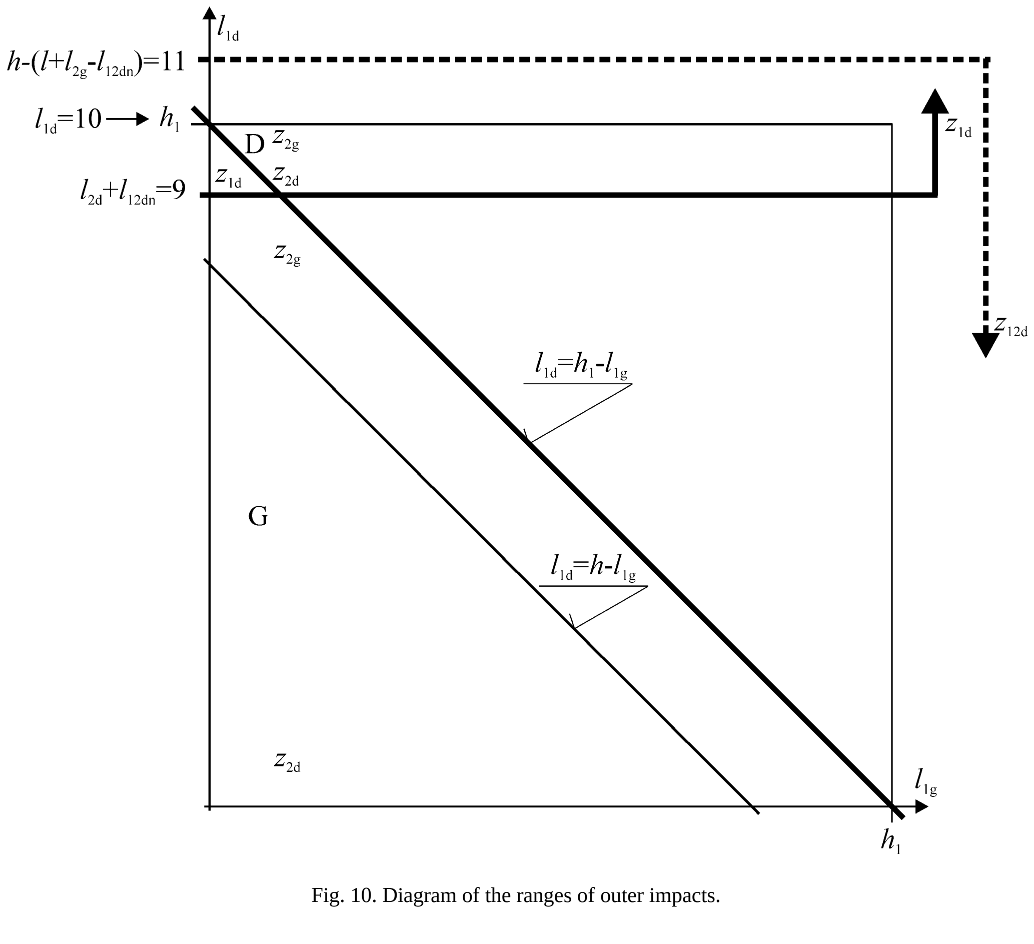

Fig. 10. Diagram of the ranges of outer impacts.

It is also worth to draw the reader's attention to the fact that in the case of a greater change in the length $l_{12\mathrm{d}}$, for instance, $z_{12\mathrm{dn}} = 7$, we make the fender $z_{1\mathrm{d}}$ passive in the system, because: $11 = l_{2\mathrm{d}} + l_{12\mathrm{dn}} \not< l_{1\mathrm{d}} = 10$. The inner fender is, however, still active, because condition $5 = (l_{1\mathrm{d}} - l_{12\mathrm{dn}}) + (l + l_{2\mathrm{g}}) < h = 8$ is fulfilled. Changing the value of the length of the inner fender $l_{12\mathrm{d}}$ to a bigger one, it will remain active and the fender $z_{1\mathrm{d}}$ will be active, if $l_{12\mathrm{dn}}$ satisfies the following inequality: $l_{1\mathrm{d}} - h + (l + l_{2\mathrm{g}}) < l_{12\mathrm{dn}} < l_{1\mathrm{d}} - l_{2\mathrm{d}}$. In Example 2 (Subsection 5.2), the changes of the parameters connected to the subsystem of the mass m_1 have been applied, thus it was not necessary to check the conditions of assembly in any of these considerations because this subsystem, due to its structure, did not have to be included in the gap h of the frame.

In general, one can conclude that the following principle holds: if the lengths of the outer fenders $l_{1\mathrm{g}}$ and $l_{1\mathrm{d}}$ aim at zero, then the inner fenders $z_{12\mathrm{g}}$ and $z_{12\mathrm{d}}$ tend to be active or, as in Example 2, if the lengths of the inner fenders $l_{12\mathrm{g}}$ and $l_{12\mathrm{d}}$ aim at infinity, then the outer fenders $z_{1\mathrm{g}}$ and $z_{1\mathrm{d}}$ tend to be passive.

6. Conclusions

The object of the investigations presented are mechanical systems with impacts with one and two degrees of freedom that have been classified previously by the authors of the present study. The models of the systems under consideration are rigid bodies that can perform a motion along a straight line without a possibility of rotations.

The geometrical conditions of assembly and the geometrical conditions of outer and inner impacts that can occur during operation have been determined. *The principle of omitting the condition*, according to which the condition that includes the dimension related to this fender is omitted automatically in the system, has been developed. This principle holds both for the conditions of assembly and for the conditions for outer and inner impacts.

In the case the conditions for impacts are written by means of non-sharp inequalities, a situation arises in which simultaneous outer impacts or simultaneous outer and inner impacts are possible in the system.

In the study, the so-called *graphic method of ranges of impacts* that allow us to answer numerous questions, for instance, if and what kind of impacts can occur in the system, has been proposed. This method also leads to some conclusions, among which the most important are as follows: (1) it is possible to design a system with a predetermined set of outer and inner impacts, (2) it is possible to identify a passive fender.

From the viewpoint of the classification of mechanical systems with impacts (Blazejczyk-Okolewska et al., 2004), the system that has a passive fender (that has been identified with the graphic method of ranges of impacts) is the so-called inapt combination and should be either eliminated through a change in the system parameters (lengths of the fenders) or we should make it an apt combination.

On the basis of the examples included in this study, it has been stated that there is a possibility to design such a mechanical system in which all outer and inner fenders will be active, and the obtained system will be the *basic impact system*, according to the classification of mechanical systems with impacts (Blazejczyk-Okolewska et al., 2004). An analysis of the dynamics of such a system will be interesting, according to the authors' opinion.

The results of the present study, based on rigid bodies without any deformation in contact points, are exactly valid for the beginning of impacts – for the first touch of impacting bodies. If the contacts between bodies are realized by means of weak stops, simultaneous contacts in different impacting pairs could occur.

The present work provides much information of the fundamental nature that broadens the scope of knowledge on the motion of mechanical systems with impacts. This information can be applied in future to calculations and design of the above-described structures. The study can be the basis for starting investigations on mechanical systems with impacts, in which the motion is multidimensional as well.

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