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# Using chaos to reduce oscillations: Experimental results

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**Abstract**

Chaotic dynamic systems are usually controlled in a way, which allows the replacement of chaotic behavior by the desired periodic motion. We give the example in which an originally regular (periodic) system is controlled in such a way as to make it chaotic. This approach based on the idea of dynamical absorber allows the significant reduction of the amplitude of the oscillations in the neighborhood of the resonance. We present experimental results, which confirm our previous numerical studies [Dąbrowski A, Kapitaniak T. Using chaos to reduce oscillations. Nonlinear Phenomen Complex Syst 2001;4(2):206–11].


## 1. Introduction

In most of chaos controlling methods the desired periodic motion replaces unpredictable chaotic behavior [2,3]. This approach is straightforward and was found to be advantageous in many physical, biological, economical and engineering systems. In [1] we presented an idea and numerical study of the controlling method in which undesirable periodic oscillations are replaced by an appropriate chaotic behavior, which improves the performance of the controlled system. In what follows we show that such an approach can be widely applied in the mechanical systems, causing the reduction of oscillations.

Recently, the great interest in oscillations' reduction and control can be seen. The authors show the huge significance of elaborating simple and effective methods of oscillations' reduction [4–8]. The most important applications of these vibration absorbers are impacting hammers, gas turbines, tubes in steam generators and their support plates, helicopter and weapon systems and ship trunks.

Simultaneously, the particular interest in vibro-impact oscillations absorbers can be seen. There are a lot of papers in which the authors show, that these kinds of absorbers are more effective and work in wider frequency range than the traditional ones [9–16]. There are still no simple methods for elaborating the constructions of vibro-impact absorbers and confirming the parameters values for which the oscillations' reduction has the practical significance. In this paper, the damper is designed in such a way that it allows the replacement of the periodic impactless oscillations with the large amplitude, by the chaotic impact motion with very small amplitude. We give the numerical and experimental evidence of the effectiveness of our controlling method.

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## 2. Dynamics of the absorber system

The physical model of the system is shown in Fig. 1. The system consists of three oscillators. The external harmonic force $F\sin\omega t$ excites the main oscillator (mass $m$ suspended on the spring with stiffness coefficient $k$, and viscotic damper with damping coefficient $c$). The system of the absorbers is designed in such a way as to control oscillations of this oscillator. The controlled oscillator is joined with the classical dynamical absorber (mass $m_1$ suspended on the spring with stiffness coefficient $k_1$ and viscotic damper with damping coefficient $c_1$). This absorber is allowed to collide with the third oscillator (mass $m_2$ suspended on the spring with stiffness coefficient $k_2$ and viscotic damper $c_2$). The dynamical absorber is matched with the resonance frequency of the main oscillator in the way, that in the range of its resonance amplitude of its oscillations is decreasing to zero. It is known that this kind of a damper is effective just for a narrow range of external force frequency. For the frequencies that are lower and higher than $\eta = \omega/(k/m)^{1/2} = 1$ there exist two another resonance frequencies $\eta_1$ and $\eta_2$ as shown in Fig. 2. In order to decrease oscillations' amplitudes in the range of these frequencies, retaining the advantages of the dynamic damper, the third independent oscillator was added. This oscillator will be called the impact absorber. In the periods between the impacts dynamics of the system is described by six ordinary differential equations:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = q\sin\eta\tau - 2\gamma y - 2\gamma_1\sigma_1\beta_1(y - y_1) - x - \beta_1^2(x - x_1), \\ \dot{x}_1 = y_1, \\ \dot{y}_1 = -2\gamma_1\sigma_1\beta_1(y_1 - y) - \beta_1^2(x_1 - x), \\ \dot{x}_2 = y_2, \\ \dot{y}_2 = -2\gamma_2\sigma_2\beta_2 y_2 - \beta_2^2 x_2, \end{cases} \tag{1}$$

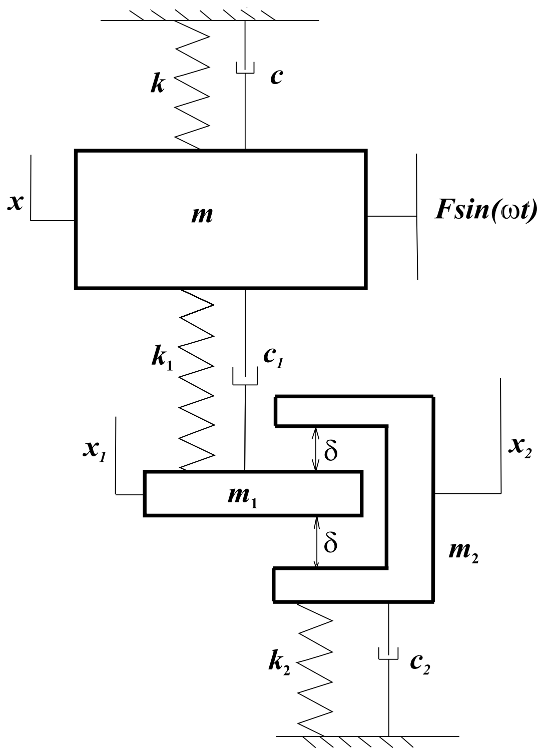



Fig. 1. Physical model of the system.

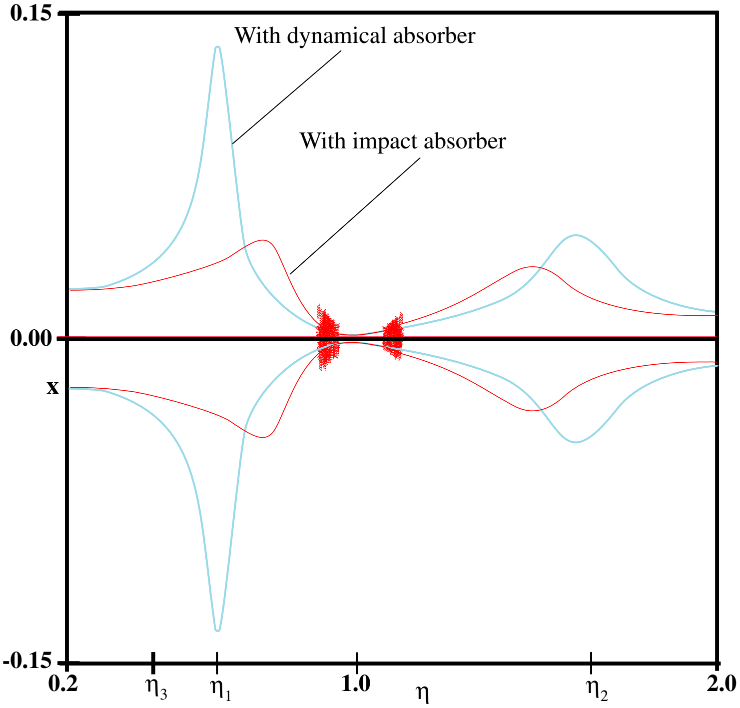



Fig. 2. Comparison between resonance curve of the system with and without impact absorber.

where

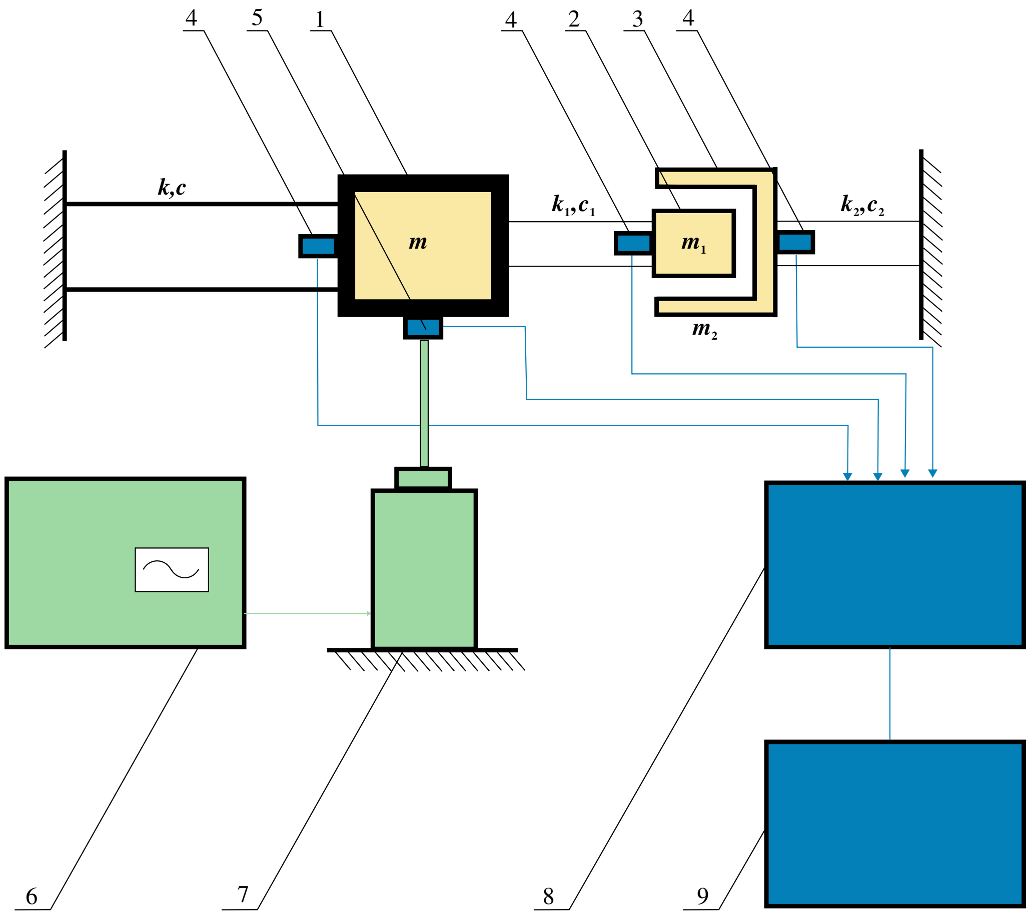
$$\begin{aligned}
&\sigma_1 = \frac{k_1}{k}, \quad \sigma_2 = \frac{k_2}{k},\\
&\beta_1 = \frac{\alpha_1}{\alpha}, \quad \beta_2 = \frac{\alpha_2}{\alpha},\\
&\alpha = \sqrt{\frac{k}{m}}, \quad \alpha_1 = \sqrt{\frac{k_1}{m_1}}, \quad \alpha_2 = \sqrt{\frac{k_2}{m_2}},\\
&\gamma = \frac{c}{2\sqrt{km}}, \quad \gamma_1 = \frac{c_1}{2\sqrt{k_1 m_1}}, \quad \gamma_2 = \frac{c_2}{2\sqrt{k_2 m_2}},\\
&\eta = \frac{\omega}{\alpha}, \quad \tau = \alpha t, \quad q = \frac{F}{k},
\end{aligned} \tag{2}$$

Impact modelling was based on Newton's law with use of the restitution coefficient $r$.

It can be seen from Fig. 1 that in the range of symmetrical clearance $\delta$, the impact absorber limits the free motion of the dynamical absorber. The value of this clearance was matched in such a way that the dynamical and impact absorbers do not collide with each other for the forcing frequency $\eta$ close to 1. It allows to take the advantages of the dynamical absorber application. In other frequencies, where the dynamical absorber oscillations' amplitude is high, the impacts occur. The energy dissipates as the result of impacts and also, depending on the relative velocity of the damper and the absorber before the impact, the energy is transmitted between the absorbers. It allows to decrease the main oscillator's amplitude in the range of frequencies $\eta_1$ and $\eta_2$. The comparison between two resonance diagrams of the main oscillator's displacement in the cases with the dynamical absorber and with both absorbers is shown in Fig. 2. The big effectiveness of the impact absorber can be seen. The oscillations' amplitude was decreased several times in the range of frequencies $\eta_1$, $\eta_2$. The very important fact is that it did not cause a big amplitude increase for other frequencies. According to the earlier declaration, for the frequency $\eta = 1$, the oscillations' amplitude was reduced to zero. The motion in almost the whole forcing frequency is regular. The chaotic motion exists only in the neighborhood of the frequency $\eta = 1$, but it is not of a big importance because of small amplitude of oscillations.

## 3. Comparison of the experiment and simulations

The main idea of the experimental system and the system's real construction are shown in Figs. 3 and 4, respectively. The considered system consists of three oscillators (controlled oscillator – 1, dynamical absorber – 2, impact absorber – 3). The external harmonic force was generated in the electrodynamic exciter – 8, which excited the controlled oscillator – 1. It was joined with the classical dynamical absorber – 2. This absorber was allowed to collide with the impact absorber $m_2$ – 3. The special kind of springs $k_1$, $k_2$, $k_3$ was allowed to obtain one dimensional motion of the oscillators. The accelerations of the main oscillator, dynamic and impact absorbers were measured using accelerometers – 4 and 5. The current intensity that was supplied to the exciter – 8 was measured in amplifer – 7. These signals were put into multi-analyser – 9, and then




1. Main system
2. Dynamic absorber
3. Impact absorber
4. Quartz Shear ICP® Accelerometers Type 353B02 PCB Piezotronics
5. Force Transducer - Type 8200 Brüel & Kjer
6. Harmonic signal generator
7. Electrodynamic Exciter - Type PR9261 firmy Philips
8. Multi-analyzer System PULSE, Type 3560 with Acquisition Front-end - Type 2825, 4-channel Input Module - Type 3022 (0,7Hz-25,6kHz).
9. Computer with software Noise and Vibration Analysis Type 7700 Brüel & Kjer

Fig. 3. Scheme of the experimental setup.

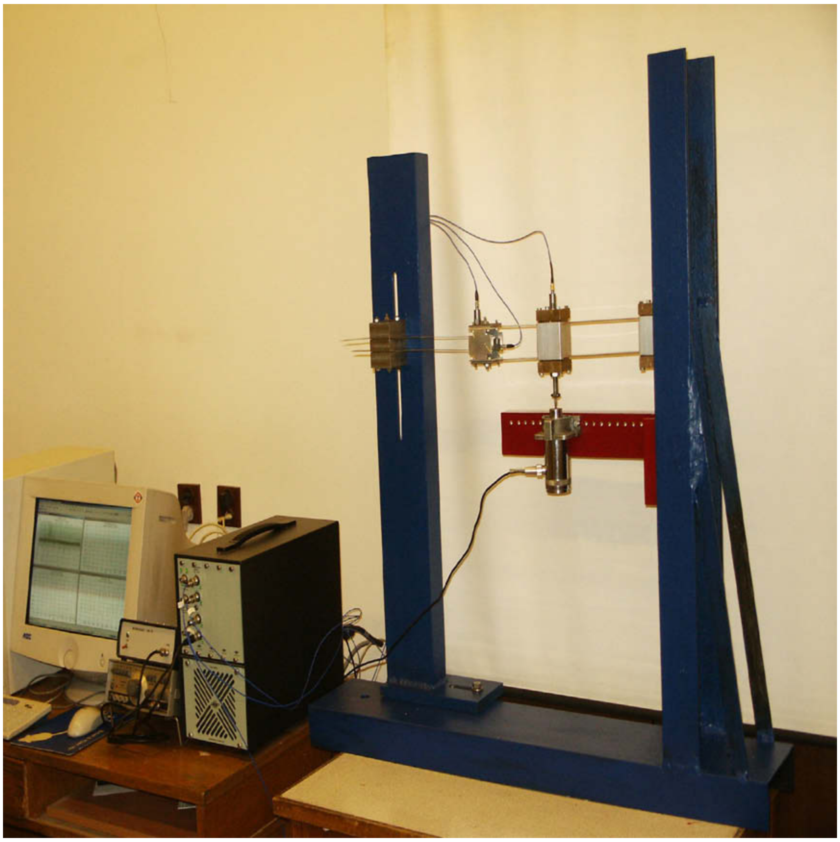


Fig. 4. View of the experimental rig.

into the computer set with the Brüel & Kjer software [17]. The signal from the accelerometers was integrated and compared with simulations results. The force amplitude was measured using the current intensity that was supplied to the exciter. The dynamic absorber was matched with the main oscillator in such a way, that in the range of the resonance frequency $\eta = 1$ amplitude the controlled system was close to zero. For the frequencies that are lower and higher than $\eta = 1$ system with dynamical absorber has two resonance frequencies $\eta_1$ and $\eta_2$. The parameters were selected in such a way, that distance between these frequencies was possibly the widest with the acceptable values of other parameters. Simultaneously frequencies $\eta_1$ and $\eta_2$ were designed to be possibly symmetrical in relation to the frequency $\eta = 1$. These preconditions allowed the most effective amplitude reduction. The parameters of the impact absorber were matched to obtain the system with possibly lower amplitude of oscillations in range of the resonance frequencies, and the automatic transition from impact into impactless motion. The value of the clearance between the impact and dynamical absorbers was matched in such a way that the dynamic and impact absorbers do not collide with each other for the forcing frequency $\eta$ close to 1, and to obtain the wide range of the impactless motion with the huge amplitude reduction We have tried to reconstruct the obtained parameters in the real laboratory system. The parameters identified in laboratory system were such as follows:

$$\begin{aligned}
&\alpha = 96.0052, \quad \gamma = 0.026210,\\
&\alpha_1 = 94.2538, \quad \gamma_1 = 7.16868\mathrm{E}-05,\\
&\alpha_2 = 84.82096, \quad \gamma_2 = 0.0462022,\\
&\mu_1 = 0.24183, \quad \mu_2 = 0.24183,\\
&\sigma_1 = 0.23308, \quad \sigma_2 = 0.18876,\\
&\beta_1 = 0.98175, \quad \beta_2 = 0.88350,\\
&\delta = 0.0015, \quad r = 0.7, \quad q = 0.000142.
\end{aligned}$$

The comparison of the simulations and experimental data is shown in Fig. 5a–c. In Fig. 5a we present resonance diagrams of the main system without absorbers. One can see a good agreement between numerical simulations and experimental data. The comparison of the simulations and experimental data for the system with dynamical absorber

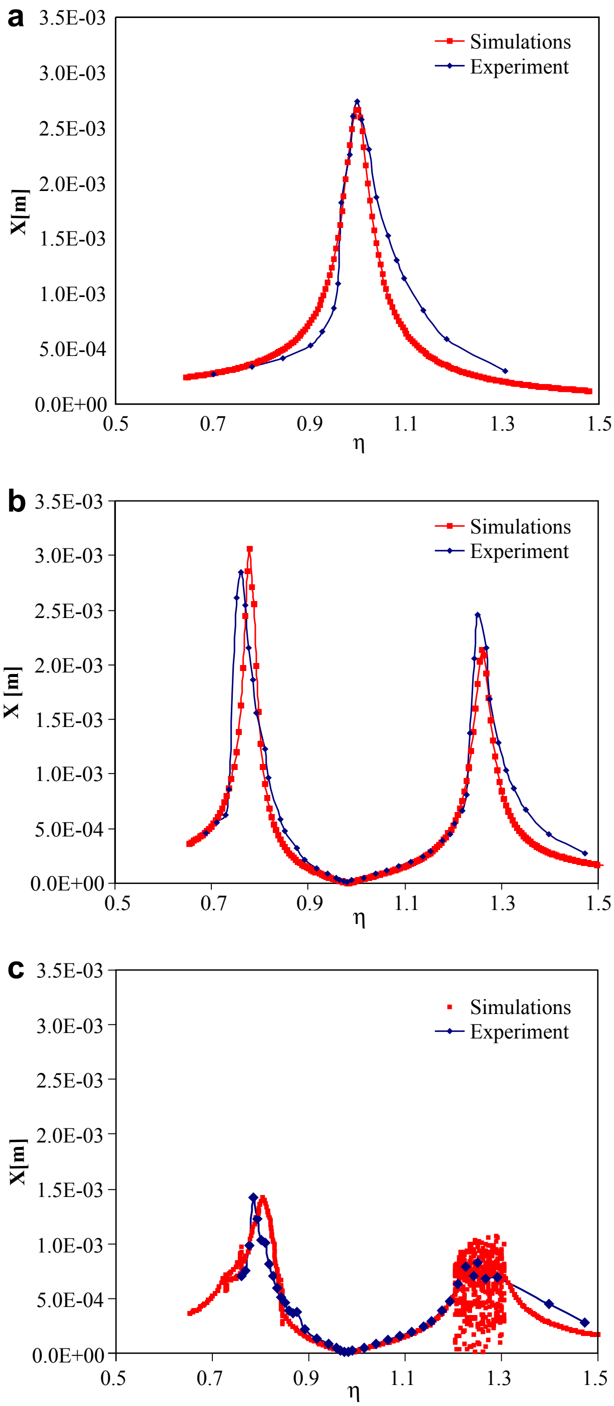



Fig. 5. Resonance curves; $\alpha = 96.0052$, $\gamma = 0.026210$, $\alpha_1 = 94.2538$, $\gamma_1 = 7.16868\text{E}{-}05$, $\alpha_2 = 84.82096$, $\gamma_2 = 0.0462022$, $\mu_1 = 0.24183$, $\mu_2 = 0.24183$, $\sigma_1 = 0.23308$, $\sigma_2 = 0.18876$, $\beta_1 = 0.98175$, $\beta_2 = 0.88350$, $\delta = 0.0015$, $r = 0.7$, $q = 0.000142$; (a) system without dampers, (b) system with dynamical damper, (c) system with dynamical and impact dampers.

(subsystem 2) is shown in Fig. 5b. One can see huge amplitude reduction in wide range between $\eta_1$ and $\eta_2$ with amplitude of oscillations close to zero in range of $\eta = 1$. Unfortunately for the frequencies $\eta_1$ and $\eta_2$ the resonance peaks are very high. The results of the experiments with impact absorber (subsystem 3) can be seen in Fig. 5c. The dynamic and impact absorbers do not collide with each other for the forcing frequency $\eta$ close to 1. The transition from impact into impactless motion is automatic, and impactless motion exists in the wide range between $\eta_1$ and $\eta_2$. For the frequencies $\eta_1$ and $\eta_2$ the amplitude was reduced several times. The maximum amplitude of oscillations for the system with impact absorber is few times lower than resonance amplitude of the system without any absorber.

## 4. Conclusions

We present the method of controlling a dynamic system in such a way that the undesirable periodic oscillations with large amplitude are replaced by chaotic motion with small amplitude. We give evidence that this approach can be especially useful in the forced mechanical systems, which are designed to operate in the neighborhood of the resonance. It has been shown that the combination of the impact and dynamic absorber could significantly reduce the amplitude of the oscillations. In almost all the frequency range the amplitude is reduced and the motion is periodic. In the neighborhood of $\eta = 1$ the impact absorber automatically has coupled with the dynamical absorber and the large amplitude periodic oscillations of the main mass are replaced by the low amplitude chaotic motion. We confirm the results of the numerical simulations experimentally.

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